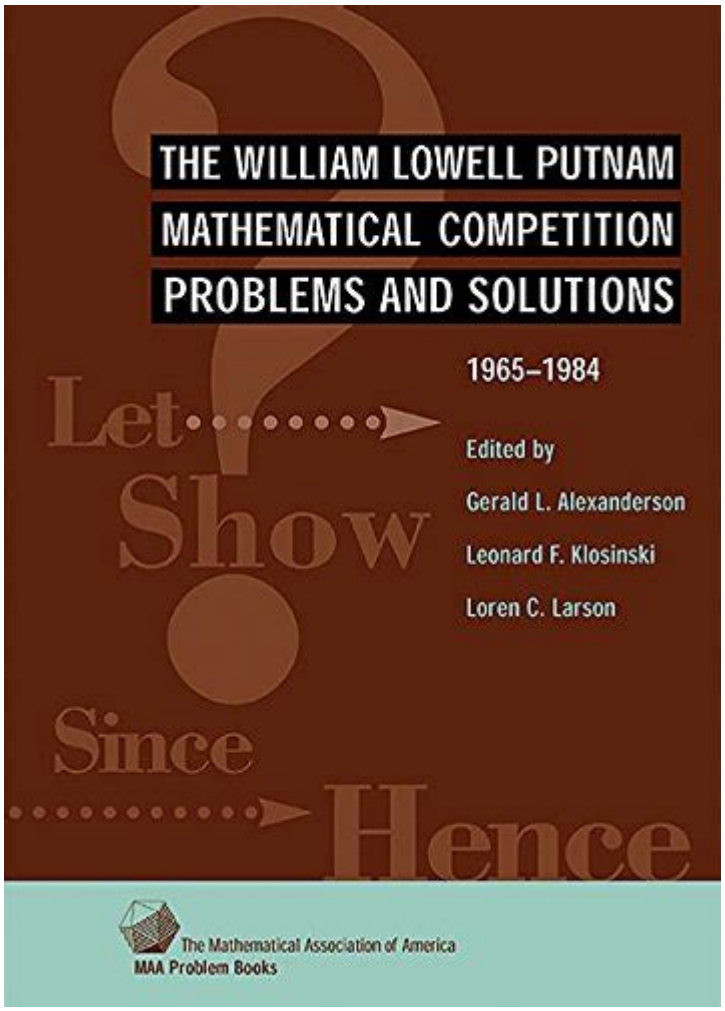
William Lowell Putnam Mathematical Competition Problems And Solutions



William Lowell Putnam Mathematical Competition problems and solutions are a cornerstone of mathematical competitions in the United States and Canada. Established in 1938, this prestigious competition is aimed at undergraduate students and has gained a reputation for its challenging problems that test not only knowledge but also creativity and problem-solving skills. In this article, we will explore the nature of the competition, examine some notable problems, and provide insights into the solutions, highlighting the strategies and methodologies used in tackling these mathematical challenges.

Overview of the William Lowell Putnam Mathematical Competition

The William Lowell Putnam Mathematical Competition is organized annually by the Mathematical Association of America (MAA). The competition consists of two 3-hour sessions, held on the first Saturday of December, where participants face 12 problems in each session, totaling 6 hours of intensive mathematical reasoning.

Format and Scoring

- Each problem is scored out of 10 points, making the maximum possible score 120 points.
- Problems are designed to cover a wide range of mathematical topics, including but not limited to:
- Algebra
- Combinatorics
- Geometry
- Number Theory
- Calculus
- The problems are known for their depth and often require innovative thinking rather than straightforward application of formulas.

Characteristics of Putnam Problems

Putnam problems are unique in several ways that contribute to their difficulty:

1. Creativity and Insight: Many problems require a deep understanding of mathematical concepts and the ability to see connections that are not immediately obvious.
2. Brevity and Clarity: Solutions must be concise and clearly articulated. Judges not only look for the correct answer but also for the logical progression of thoughts leading to that answer.
3. Diverse Topics: Participants must be well-rounded in various mathematical fields, as questions can come from any area of undergraduate mathematics.

Notable Putnam Problems and Solutions

To illustrate the nature of Putnam problems, we will look at a few notable examples from past competitions, along with their solutions.

Example Problem 1: 1985, Problem A3

Problem Statement: Let $a_1, a_2, \ldots, a_n$ be real numbers such that $a_1 + a_2 + \ldots + a_n = 0$. Show that there exist real numbers $b_1, b_2, \ldots, b_n$ such that $b_1 + b_2 + \ldots + b_n = 0$ and $b_i = a_i^2$ for $i = 1, 2, \ldots, n$.

Solution:
The problem asks for real numbers b_i that sum to zero while being squares of a_i. By the properties of squares, we realize that b_i will always be non-negative. Thus, if $n > 1$, we can choose one a_i to be negative and another to be positive, ensuring that their squares will not balance to zero. Hence, we conclude that such b_i exist when $n \geq 2$.

Example Problem 2: 2003, Problem B6

Problem Statement: Let $f(x)$ be a polynomial of degree 2003 such that $f(x) = f(1-x)$ for all x. Find the maximum possible degree of $f(x)$ if $f(0) = 1$.

Solution:
Given that $f(x) = f(1-x)$, we deduce that f is symmetric about $x = \frac{1}{2}$. This implies that all odd degree terms must cancel out, leaving us with an even polynomial. The highest degree of f is thus limited to 2002, as any odd degree would violate the symmetry. Consequently, we conclude that the maximum degree is 2002.

Strategies for Solving Putnam Problems

To succeed in the Putnam competition, students should develop a range of strategies and techniques:

1. Understanding the Problem

- Read Carefully: Take your time to understand the problem statement fully. Misinterpretation can lead to incorrect approaches.
- Identify Key Ideas: Look for patterns or special cases that might simplify the problem.

2. Structured Approach to Solutions

- Work through Examples: Before diving into abstraction, consider specific cases which can lead to general insights.
- Break It Down: Divide the problem into smaller, manageable parts that can be tackled individually.

3. Collaboration and Discussion

- Form Study Groups: Discussing problems with peers can expose you to different perspectives and methodologies.
- Review Past Problems: Analyzing solutions to previous Putnam problems can provide insight into common techniques and approaches.

4. Practice and Time Management

- Mock Competitions: Simulate the pressure of the competition environment by timing yourself on past problems.
- Balanced Preparation: While it’s crucial to practice problems, ensure you also dedicate

time to review theory and concepts.

Conclusion

The William Lowell Putnam Mathematical Competition presents a rigorous challenge for aspiring mathematicians and problem solvers. The problems and solutions reflect a depth of creativity and understanding that goes beyond rote learning. For participants, success in the Putnam competition involves not just mathematical knowledge but also strategic thinking, collaboration, and extensive practice.

As students continue to engage with the competition, they will not only hone their mathematical skills but also develop a profound appreciation for the beauty and intricacy of mathematics. Whether one is a seasoned competitor or a newcomer looking to challenge themselves, the Putnam competition offers an invaluable opportunity for growth and discovery in the world of mathematics.

Frequently Asked Questions

What is the William Lowell Putnam Mathematical Competition?

The William Lowell Putnam Mathematical Competition is an annual mathematics competition for undergraduate students in the United States and Canada. It is known for its challenging problems that test creativity, problem-solving skills, and mathematical knowledge.

How are the problems in the Putnam competition structured?

The Putnam competition consists of two 3-hour sessions, each containing 6 problems. The problems are designed to be difficult and require a deep understanding of mathematics, often involving creative approaches and advanced concepts.

Where can I find past Putnam competition problems and solutions?

Past Putnam competition problems and their solutions can be found on various mathematics websites, including the official Putnam competition website, university math department pages, and mathematical forums dedicated to problem-solving.

What topics are typically covered in the Putnam problems?

Putnam problems cover a wide range of topics including algebra, combinatorics, calculus, number theory, and geometry. The problems often require interdisciplinary knowledge

and innovative problem-solving techniques.

How can one prepare effectively for the Putnam competition?

Effective preparation for the Putnam competition includes studying past problems and solutions, participating in math clubs or study groups, taking advanced mathematics courses, and practicing problem-solving regularly to build intuition and familiarity with various topics.

What is the significance of the Putnam competition in the mathematical community?

The Putnam competition is highly regarded in the mathematical community as it identifies and nurtures mathematical talent among undergraduates. It also enhances problem-solving skills and promotes a culture of mathematical excellence, often leading to opportunities in academia and industry.

William Lowell Putnam Mathematical Competition Problems And Solutions

为什么英语人名「William」常昵称作「Bill」？ - 知乎

很多的人名在这一时期都被通过替换首个辅音的方法制造出来了。例如Molly (Mary)>Polly, Rob (Robert)>Bob。Behind the Name: Meaning, Origin and History of the Name Bill Why do people named William get called Bill? Rhyming slang 小知识 --- Cockney Rhyming Slang 伦敦押韵俚语 知乎用户 2 人赞同了该回答

如何看待 2025 年 6 月 30 日小约翰可汗最新一期视频中的莫里斯· …

真实的历史没有故事中的温情脉脉。 1964年，苏联和美共（CPUSA）察觉到资金流被监控了，很快也怀疑到了视频的主角Morris Childs身上，他和KGB的会面被临时取消，一下子陷入了危险当中。为了保护Morris，FBI制定了一项暗渡陈仓的计划，诬陷CPUSA在纽约的负责人William Albertson是FBI线人。由于“顺利找到了 …

威廉（**William**）的昵称为什么叫比尔（**Bill**）？这其中有什么渊 **…**

Oct 31, 2015 · 谢邀！ Why do people named William get called Bill? 这里有比较详细的分析。补充一下：W这个字母，在德语字母表里的发音，读起来就是“Double V”，也就是“两个V”，可见W是从V叠加而来的，而V在 西班牙语 里的大部分时候的发音就是 [b]。这些欧洲语言都相互有影响有渊源，所以可能“Bill”的来源和这个 …

威廉玛丽学院国内认可度怎么样？ - 知乎

熟悉美国留学的应该都知道WM叭，很有名的文理学院教育体制，不过WM实话说规模已经不算传统意义上的文理学院了 华盛顿杰斐逊 都是alumni QS 这个排名乐呵一下就行，Emory 也很惨哈哈。不过确实有一些HR会去卡这个，而且是无脑卡这个，Top10券商HR也干这事，没办法 人就是卡北美六大人都能招到不少 …

为什么**Fundamental of power electronics**这本书没有人进行正式的 **…**

David Middlebrook终身成就奖，2023年度IEEE William E. Newell电力电子奖章。他是电力电子领域经典教材《Fundamentals of Power Electronics》的作者。

怎么区分外国人的名字 **first name**与**last name?**_百度知道

怎么区分外国人的名字 first name与last name?first name是名，Leszek是名，Godzik是last name是姓。姓名排列次序是名在姓前，一般由教名、名字和姓三部分组成，在正文中要按此顺序书写，并且名字可缩写，姓不能缩

为什么**Bill**是**William**的昵称？_百度知道

Jul 17, 2007 · 为什么Bill是William的昵称？William 威廉 来源：法国。涵意：一位强而有力的战士或保护者。而bill这个名字在古希腊神话中，是指战神的后代比尔，也是勇悍的士兵啊 所以作为william的昵称了吧。英文名一般都有昵称，

如何评价威廉. 叶芝（William Yeats)的 "the second coming"？

事实上，第19行的“二十世纪”和对伯利恒的提及似乎证实了这首诗与基督教的联系，因为两者都提到了耶稣基督的诞生。【参考资料】William Butler Yeats（威廉·巴特勒·叶芝)《The Second Coming》《第二次降临》电子书和网络公开资料。

莎士比亚的四大悲剧和四大喜剧分别是什么？_百度知道

1、威廉·莎士比亚（William Shakespeare 1564–1616)，是英国文学史和戏剧史上最杰出的诗人和剧作家，也是西方文艺史上最杰出的作家之一，全世界卓越的剧作家之一。他的剧本被翻译成世界各国几乎所有正在通用的语言，其作品的地位和影响力与《圣经》并驾齐驱。

对一个陌生的英文名字，如何快速确定哪个是姓哪个是名？ **-** 知乎

这里我以美国人的名字为例，在美国呢，人们习惯于把自己的名字 (first name)放在前,姓放在后面 (last name). 这也就是为什么叫first name或者last name的原因（根据位置摆放来命名的）。比如美剧生活大爆炸的谢耳朵（Sheldon Cooper)，他姓Cooper，名字叫Sheldon. 我觉得这样的习惯可能是因为名字是一个人的个人 …

为什么英语人名「**William**」常昵称作「**Bill**」？ **-** 知乎

很多的人名在这一时期都被通过替换首个辅音的方法制造出来了。例如Molly (Mary)>Polly, Rob (Robert)>Bob。Behind the Name: Meaning, Origin and History of the Name Bill Why do …

如何看待 **2025** 年 **6** 月 **30** 日小约翰可汗最新一期视频中的莫里斯· **…**

真实的历史没有故事中的温情脉脉。 1964年，苏联和美共（CPUSA）察觉到资金流被监控了，很快也怀疑到了视频的主角Morris Childs身上，他和KGB的会面被临时取消，一下子陷入了危险 …

威廉（**William**）的昵称为什么叫比尔（**Bill**）？这其中有什么渊 **…**

Oct 31, 2015 · 谢邀！ Why do people named William get called Bill? 这里有比较详细的分析。补充一下：W这个字母，在德语字母表里的发音，读起来就是“Double V”，也就是“两个V”，可 …

威廉玛丽学院国内认可度怎么样？ - 知乎

熟悉美国留学的应该都知道WM叭，很有名的文理学院教育体制，不过WM实话说规模已经不算传统意义上的文理学院了 华盛顿杰斐逊 都是alumni QS 这个排名乐呵一下就行，Emory 也很惨 …

为什么Fundamental of power electronics这本书没有人进行正式的 …

David Middlebrook终身成就奖，2023年度IEEE William E. Newell电力电子奖章。他是电力电子领域经典教材《Fundamentals of Power Electronics》的作者。

怎么区分外国人的名字 **first name**与**last name?**_百度知道

怎么区分外国人的名字 first name与last name?first name是名，Leszek是名，Godzik是last name是姓。姓名排列次序是名在姓前，一般由教名、名字和姓三部分组成，在正文中要按此 …

为什么Bill是William的昵称？_百度知道

Jul 17, 2007 · 为什么Bill是William的昵称？William 威廉 来源：法国。涵意：一位强而有力的战士或保护者。而bill这个名字在古希腊神话中，是指战神的后代比尔，也是勇悍的士兵啊 所以作 …

如何评价威廉. 叶芝（*William Yeats)*的 "*the second coming*"?
事实上，第19行的“二十世纪”和对伯利恒的提及似乎证实了这首诗与基督教的联系，因为两者都提到了耶稣基督的诞生。【参考资料】William Butler Yeats (威廉·巴特勒·叶芝)《The Second ...

莎士比亚的四大悲剧和四大喜剧分别是什么？_百度知道
1、威廉·莎士比亚（William Shakespeare 1564–1616)，是英国文学史和戏剧史上最杰出的诗人和剧作家，也是西方文艺史上最杰出的作家之一，全世界卓越的剧作家之一。他的剧本被翻译 ...

对一个陌生的英文名字，如何快速确定哪个是姓哪个是名？ - 知乎
这里我以美国人的名字为例，在美国呢，人们习惯于把自己的名字 (first name)放在前,姓放在后面 (last name). 这也就是为什么叫first name或者last name的原因（根据位置摆放来命名的）。比 ...

Explore the William Lowell Putnam Mathematical Competition problems and solutions. Enhance your problem-solving skills and prepare for success. Learn more!

Back to Home