

# William Lowell Putnam Mathematical Competition

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William Lowell  
**PUTNAM**  
Mathematical Competition

Problems for  
Session A

## The 81st William Lowell Putnam Mathematical Competition

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A1. How many positive integers  $N$  satisfy all of the following three conditions?

- (i)  $N$  is divisible by 2020.
- (ii)  $N$  has at most 2020 decimal digits.
- (iii) The decimal digits of  $N$  are a string of consecutive ones followed by a string of consecutive zeros.

A2. Let  $k$  be a nonnegative integer. Evaluate

$$\sum_{j=0}^k 2^{k-j} \binom{k+j}{j}.$$

A3. Let  $a_0 = \pi/2$ , and let  $a_n = \sin(a_{n-1})$  for  $n \geq 1$ . Determine whether

$$\sum_{n=1}^{\infty} a_n^2$$

converges.

A4. Consider a horizontal strip of  $N+2$  squares in which the first and the last square are black and the remaining  $N$  squares are all white. Choose a white square uniformly at random, choose one of its two neighbors with equal probability, and color this neighboring square black if it is not already black. Repeat this process until all the remaining white squares have only black neighbors. Let  $w(N)$  be the expected number of white squares remaining. Find

$$\lim_{N \rightarrow \infty} \frac{w(N)}{N}.$$

A5. Let  $a_n$  be the number of sets  $S$  of positive integers for which

$$\sum_{k \in S} F_k = n,$$

where the Fibonacci sequence  $(F_k)_{k \geq 1}$  satisfies  $F_{k+2} = F_{k+1} + F_k$  and begins  $F_1 = 1$ ,  $F_2 = 1$ ,  $F_3 = 2$ ,  $F_4 = 3$ . Find the largest integer  $n$  such that  $a_n = 2020$ .

A6. For a positive integer  $N$ , let  $f_N$  be the function defined by

$$f_N(x) = \sum_{n=0}^N \frac{N+1/2-n}{(N+1)(2n+1)} \sin((2n+1)x).$$

Determine the smallest constant  $M$  such that  $f_N(x) \leq M$  for all  $N$  and all real  $x$ .

**The William Lowell Putnam Mathematical Competition** is one of the most prestigious and challenging mathematics competitions for undergraduate students in the United States and Canada. Established in 1938, the competition aims to foster a love for mathematics and to identify and promote outstanding mathematical talent among students. Each year, thousands of students participate, solving complex problems that test their creativity, analytical thinking, and problem-solving abilities. This article delves into the history, structure, significance, preparation strategies, and impact of the Putnam Competition on the mathematical community.

# History of the Putnam Competition

The Putnam Competition was established by William Lowell Putnam, a successful businessman and philanthropist who had a strong interest in mathematics. The competition was initiated to honor his mother, Elizabeth Lowell Putnam, and it was designed to encourage college students to engage with mathematics more deeply.

- The first competition took place on December 10, 1938.
- Initially, the competition was only open to students from colleges and universities in the United States and Canada.
- Over the years, the competition has grown in popularity, with participation from hundreds of institutions and thousands of students annually.

## Structure of the Competition

The Putnam Competition is held on the first Saturday in December and consists of two 3-hour sessions. Each session features a set of six problems, making a total of twelve problems for the competition.

## Competition Format

- Duration: 6 hours total (2 sessions of 3 hours each).
- Problems: 12 problems total (6 in each session).
- Scoring:
  - Each problem is worth up to 10 points, with a maximum possible score of 120 points.
  - Problems range from straightforward computations to complex theorems requiring deep insights.

## Problem Types

The problems in the Putnam Competition are known for their creativity and variety. They often cover a range of topics, including:

1. Algebra
2. Calculus
3. Combinatorics
4. Geometry
5. Number Theory
6. Analysis

Participants are often required to use innovative reasoning and advanced mathematical techniques to solve these problems.

## Significance of the Competition

The Putnam Competition holds substantial significance in the academic and professional realms for several reasons.

## **Encouragement of Mathematical Exploration**

- The competition encourages students to explore advanced topics beyond their coursework.
- It fosters a sense of curiosity and creativity, prompting students to think critically and independently.

## **Identification of Talent**

- The competition serves as a platform for identifying exceptionally talented students, many of whom go on to pursue advanced studies in mathematics and related fields.
- Top performers are often recruited by graduate programs and research institutions due to their demonstrated problem-solving skills.

## **Impact on Careers**

- Success in the Putnam Competition can greatly enhance a student's resume and academic profile.
- Many former participants have attained significant positions in academia, industry, and research, contributing to advancements in mathematics and technology.

## **Preparation Strategies for Participants**

Preparing for the Putnam Competition requires dedication, practice, and strategic planning. Here are several effective strategies that aspiring participants can use to enhance their performance.

## **Study Resources**

- Books: Numerous books are available that focus on problem-solving techniques and competition-style questions, such as:
  - "The Art and Craft of Problem Solving" by Paul Zeitz
  - "Putnam and Beyond" by Titu Andreescu and Zuming Feng
- Online Courses: Various online platforms offer courses specifically designed to prepare students for mathematics competitions.
- Past Papers: Reviewing past Putnam problems helps familiarize participants with the type and style of questions asked.

## **Practice Groups**

- Forming or joining a study group can provide motivation and support.
- Collaborative problem-solving allows participants to learn from each other and develop diverse approaches to difficult problems.

## **Regular Problem Solving**

- Regular practice through solving problems from previous competitions or other mathematical challenges is crucial.
- Allocating specific times each week for focused problem-solving sessions can greatly improve skills and confidence.

## **Impact on the Mathematical Community**

The Putnam Competition has a profound impact on the mathematical community, shaping the landscape of undergraduate mathematics education and research.

## **Influence on Curriculum**

- Many universities have adapted their curricula to incorporate problem-solving techniques and topics that align with the competition's focus.
- The competition has encouraged institutions to offer more advanced courses, workshops, and seminars in mathematics.

## **Community and Networking**

- The competition fosters a strong sense of community among participants, alumni, and educators.
- Networking opportunities arise through various mathematical events, conferences, and gatherings related to the Putnam Competition.

## **Conclusion**

The William Lowell Putnam Mathematical Competition stands as a hallmark of mathematical excellence and a beacon for students aspiring to deepen their understanding of mathematics. Its rigorous problems challenge students to think critically and creatively, while its rich history and significant impact on education and professional opportunities underscore the importance of mathematical competitions in cultivating talent. As the competition continues to evolve, it remains a vital component of the mathematical landscape, inspiring generations of young mathematicians to explore the beauty and depth of mathematics. For those willing to take the challenge, the Putnam Competition offers not just a test of skill, but a journey of intellectual discovery.

## **Frequently Asked Questions**

### **What is the William Lowell Putnam Mathematical Competition?**

The William Lowell Putnam Mathematical Competition is an annual mathematics

competition for undergraduate students in the United States and Canada, known for its challenging problems and prestigious reputation.

## **Who can participate in the Putnam Competition?**

The competition is open to undergraduate students enrolled at institutions in the United States and Canada, with no restrictions on the number of participants from each institution.

## **How is the Putnam Competition structured?**

The competition consists of two 3-hour sessions, each featuring six problems that participants must solve, for a total of twelve problems.

## **What is the scoring system for the Putnam Competition?**

Each problem is worth 10 points, with partial credit possible, leading to a maximum score of 120 points. Scores are typically based on the number of correct answers and the quality of solutions.

## **What types of problems are featured in the Putnam Competition?**

Problems in the Putnam Competition cover a wide range of mathematical topics, including algebra, combinatorics, geometry, number theory, and calculus, often requiring creative and non-standard approaches.

## **How is the Putnam Competition viewed in the mathematical community?**

The Putnam Competition is highly regarded in the mathematical community as a prestigious event that showcases mathematical talent, and performing well can enhance a student's academic and professional opportunities.

## **When is the Putnam Competition held?**

The Putnam Competition is typically held on the first Saturday of December each year.

## **What resources are recommended for preparing for the Putnam Competition?**

Recommended resources include past Putnam problems and solutions, books on problem-solving techniques, mathematics competitions training materials, and participation in math clubs or study groups.

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