What Is Philosophy Of Mathematics

THE PHILOSOPHY OF MATHEMATICS EDUCATION: AN OVERVIEW[1]

Paul Ernest

ABSTRACT

This chapter offers an overview of the philosophy of mathematics education. This sub-field is characterised in both narrow and broad terms, concerning the aims of mathematics education and all philosophical aspects of research in mathematics education, respectively. The sub-field is also explored in terms of its questions and practices, which can be called a bottom-up perspective, as well as in terms of the applications of branches of philosophy to mathematics education, which might be called a top-down perspective. From the bottom-up one can characterize the area in terms of questions, and I have asked: What are the aims and purposes of teaching and learning mathematics? What is mathematics? How does mathematics relate to society? What is learning mathematics? What is mathematics teaching? What is the status of mathematics education as knowledge field? In characterizing the sub-field from a 'top down' perspective I look briefly at the contributions of ontology and metaphysics, aesthetics, epistemology and learning theory, social philosophy, ethics, and the research methodology of mathematics education. This reveals both how rich and deep the contributions of philosophy are to the theoretical foundations of our field of study. But even these different approaches leave many questions unanswered. For example: what are the responsibilities of mathematics and what is the responsibility of our own subfield, the philosophy of mathematics education? I conclude that the role of the philosophy of mathematics education is to analyse, question, challenge, and critique the claims of mathematics education practice, policy and research.

INTRODUCTION: WHAT IS THE PHILOSOPHY OF MATHEMATICS EDUCATION?

In the past quarter century the philosophy of mathematics education has emerged as a loosely defined area of research, primarily concerned with the philosophical aspects of mathematics education. This chapter aims to briefly map out some of its terrain, and attempt a synoptic vision of the breadth and depth of this area. This will prepare the ground for the more detailed and specific enquiries in the chapters that follow. This task is made all the more urgent as the question of what constitutes the philosophy of mathematics education is not without ambiguity and multiple answers. For example, is *the* philosophy of mathematics education a particular, specific approach to mathematics teaching and learning or to mathematics education research? The definite article might be taken to imply a claim for the definitiveness of the account, namely, that a unique philosophy, rationale or direction is proposed. This is not what is intended here, and 'the' is meant to indicate a definite area of enquiry, a specific domain, not a single fixed ideological perspective. Thus, *the* philosophy of mathematics education is not a dominant interpretation but rather a particular, if not completely defined, area of study and investigation, a sub-specialism within mathematics education.

1 This is a draft of the chapter: Paul Ernest, The Philosophy of Mathematics Education: An Overview. In Paul Ernest (Ed.) *The Philosophy of Mathematics Education Today*, Springer, 2018, Pages 13-37. https://link.springer.com/book/10.1007/978-3-319-77760-3

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Philosophy of mathematics is a branch of philosophy that contemplates the nature and foundations of mathematics. It examines the assumptions, foundations, and implications of mathematical concepts and practices, seeking to understand the essence of mathematical truth and existence. This discipline intersects various fields, including logic, epistemology, and metaphysics, raising fundamental questions about the nature of mathematical objects, the status of mathematical statements, and the role of mathematics in the broader context of human knowledge. By delving into these philosophical inquiries, the philosophy of mathematics not only illuminates the underpinnings of mathematical thought but also reveals the inherent complexities and paradoxes within this seemingly straightforward discipline.

Historical Background

The philosophy of mathematics has a rich and varied history, from ancient philosophical discourses to contemporary debates. Key historical milestones include:

1. Ancient Philosophies

- Plato: Plato regarded mathematics as a means to access the realm of forms, asserting that mathematical entities exist independently of human thought.
- Aristotle: Aristotle focused on the empirical basis of mathematics, emphasizing its connection to the physical world and its application in understanding reality.

2. Medieval and Renaissance Thought

- Islamic Mathematicians: Scholars such as Al-Khwarizmi and Al-Kindi contributed significantly to mathematical logic and the foundations of algebra.
- Descartes: In the 17th century, René Descartes introduced Cartesian coordinates, bridging algebra and geometry, and reinforcing the importance of mathematical reasoning.

3. Modern Developments

- Kant: Immanuel Kant posited that mathematics is a synthetic a priori knowledge, meaning that it is both informative and necessary for understanding the physical world.
- Frege, Russell, and Hilbert: In the late 19th and early 20th centuries, these philosophers and mathematicians sought to establish a rigorous logical foundation for mathematics, culminating in the development of formal systems.

Key Questions in the Philosophy of Mathematics

The philosophy of mathematics seeks to address several key questions:

1. What are Mathematical Objects?

This question revolves around the ontology of mathematical entities:

- Platonism: Proposes that mathematical objects exist in an abstract realm independent of human minds.
- Nominalism: Argues that mathematical entities do not exist; instead, they are merely names or labels for collections of objects and concepts.
- Formalism: Suggests that mathematics is a manipulation of symbols according to

specified rules, without necessarily referring to any external reality.

2. What is the Nature of Mathematical Truth?

This inquiry explores the epistemological status of mathematical statements:

- Logical Truth: Some philosophers argue that mathematical truths are derived from logical axioms and theorems.
- Intuitionism: Founded by L.E.J. Brouwer, this view posits that mathematical truths are constructed by the human mind and are not discovered.
- Constructivism: Holds that mathematical objects are constructed through mental processes and that existence is tied to our ability to construct them.

3. How Do We Acquire Mathematical Knowledge?

This question addresses the sources of our mathematical knowledge:

- Empiricism: Suggests that mathematical knowledge stems from sensory experience and observation.
- Rationalism: Argues that mathematical knowledge is innate and can be deduced through reason and intellectual insight.

Major Philosophical Perspectives in Mathematics

Various philosophical perspectives offer distinct interpretations of mathematics:

1. Platonism

Platonism is one of the most influential views in the philosophy of mathematics. It posits that:

- Mathematical objects exist independently of human thought and language.
- These objects have a timeless and unchanging nature.
- Mathematical truths are discovered rather than invented.

Platonists argue that the universality and objectivity of mathematical truths support the existence of an abstract mathematical realm.

2. Nominalism

Nominalism challenges the existence of abstract mathematical entities, asserting that:

- Mathematical statements can be understood without invoking abstract objects.
- Mathematics is a language or a system of symbols that helps describe relationships and patterns in the physical world.
- The focus is on the practical use of mathematics rather than its metaphysical implications.

Nominalists emphasize that mathematics is a human construct, with its validity rooted in applications and utility.

3. Formalism

Formalism, primarily associated with mathematician David Hilbert, suggests that:

- Mathematics is a collection of formal systems based on axioms and rules of inference.
- The truth of mathematical statements is determined by their syntactical structure rather than their meaning.
- Mathematics does not have to refer to any external reality; it is a game played with symbols.

Formalism emphasizes the importance of consistency and completeness in mathematical systems.

4. Intuitionism

Intuitionism, championed by L.E.J. Brouwer, posits that:

- Mathematics is a mental construct, and mathematical truths are not objective realities.
- Mathematical objects are created by the mathematician’s mind through intuition.
- The emphasis is on constructive proof, where existence is tied to our ability to construct examples.

This perspective challenges classical mathematics, particularly in areas like set theory and the law of excluded middle.

Contemporary Issues in the Philosophy of Mathematics

The philosophy of mathematics continues to evolve and address contemporary issues:

1. The Role of Technology

With advancements in computing and artificial intelligence, new questions arise about:

- The role of algorithms in mathematics and whether they can produce genuine mathematical knowledge.
- The relationship between human intuition and machine-generated proofs.

2. Mathematical Pluralism

Mathematical pluralism suggests that:

- There are multiple valid approaches and frameworks within mathematics.
- Different mathematical systems can coexist and provide unique insights.

This perspective encourages a broader understanding of mathematical practices and their applicability across various domains.

3. The Applicability of Mathematics

The applicability of mathematics in the natural sciences raises inquiries about:

- Whether mathematical structures genuinely represent physical phenomena or if they are merely useful tools.
- The status of mathematical models in scientific theories and their implications for understanding reality.

Conclusion

The philosophy of mathematics is a vibrant field that explores the fundamental nature of mathematical thought and its implications for knowledge, existence, and reality. As it grapples with enduring questions about the existence of mathematical objects, the nature of mathematical truth, and the sources of mathematical knowledge, it continues to stimulate rich discussions among philosophers, mathematicians, and scientists alike. The interplay between mathematical practices and philosophical inquiry not only deepens our understanding of mathematics but also enriches our appreciation of its profound impact on human knowledge and the way we conceptualize the world. In an era marked by rapid technological advancement and interdisciplinary collaboration, the philosophy of mathematics remains a crucial domain for examining the foundations and implications of one of humanity's most powerful intellectual achievements.

Frequently Asked Questions

What is the philosophy of mathematics?

The philosophy of mathematics is a branch of philosophy that studies the nature and foundational aspects of mathematics, including its objects, methods, and implications.

What are the main questions addressed in the philosophy of mathematics?

Key questions include: What is the nature of mathematical objects? Are mathematical truths discovered or invented? What is the relationship between mathematics and the physical world?

What are some major schools of thought in the philosophy of mathematics?

Major schools include Platonism, which asserts that mathematical objects exist independently; Formalism, which focuses on the manipulation of symbols; and Intuitionism, which emphasizes mathematical constructions.

How does the philosophy of mathematics relate to other areas of philosophy?

It intersects with metaphysics, epistemology, and logic, raising questions about existence, knowledge, and the nature of truth in mathematical contexts.

What role does logic play in the philosophy of mathematics?

Logic is foundational in the philosophy of mathematics, as it provides the framework for formal proofs and reasoning that underlie mathematical theories.

Can mathematical objects exist independently of human thought?

This is a debated topic; Platonists argue they do, while nominalists contend that mathematical objects are merely linguistic constructs without independent existence.

How does the philosophy of mathematics impact the practice of mathematics?

Philosophical perspectives can influence the interpretation of mathematical results, the acceptance of new theories, and the methods used in mathematical research.

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