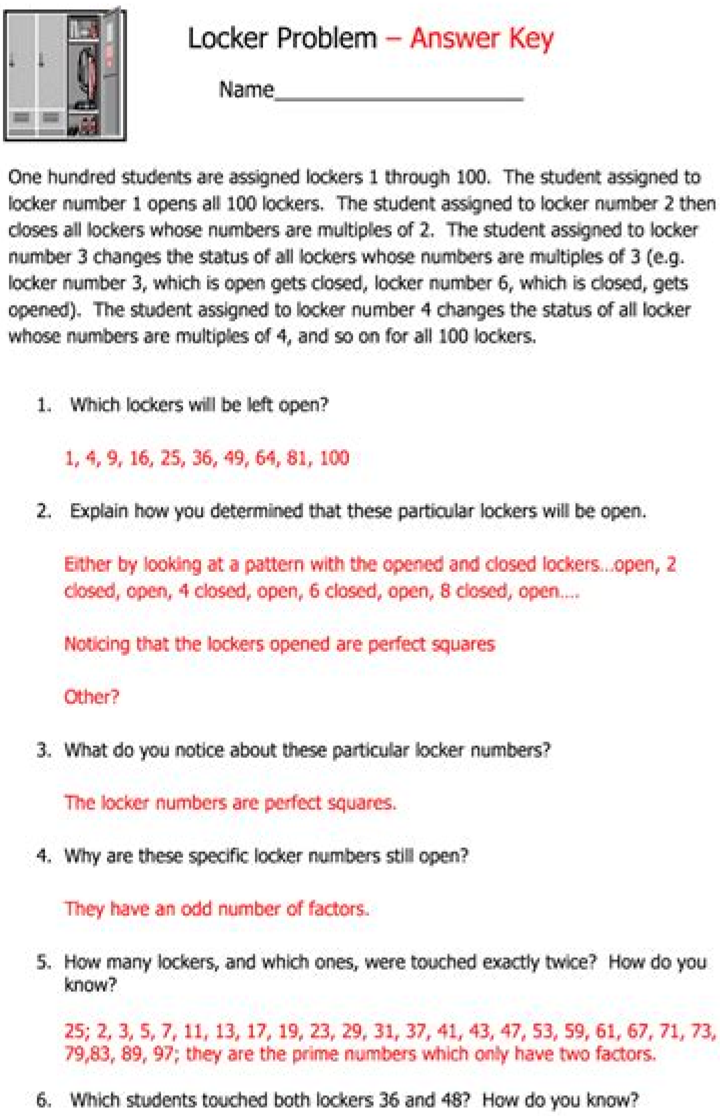
The Locker Problem Answer Key



Locker Problem – Answer Key

Name____________________

One hundred students are assigned lockers 1 through 100. The student assigned to locker number 1 opens all 100 lockers. The student assigned to locker number 2 then closes all lockers whose numbers are multiples of 2. The student assigned to locker number 3 changes the status of all lockers whose numbers are multiples of 3 (e.g. locker number 3, which is open gets closed, locker number 6, which is closed, gets opened). The student assigned to locker number 4 changes the status of all locker whose numbers are multiples of 4, and so on for all 100 lockers.

1. Which lockers will be left open?

   1, 4, 9, 16, 25, 36, 49, 64, 81, 100

2. Explain how you determined that these particular lockers will be open.

   Either by looking at a pattern with the opened and closed lockers…open, 2 closed, open, 4 closed, open, 6 closed, open, 8 closed, open….

   Noticing that the lockers opened are perfect squares

   Other?

3. What do you notice about these particular locker numbers?

   The locker numbers are perfect squares.

4. Why are these specific locker numbers still open?

   They have an odd number of factors.

5. How many lockers, and which ones, were touched exactly twice? How do you know?

   25; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79,83, 89, 97; they are the prime numbers which only have two factors.

6. Which students touched both lockers 36 and 48? How do you know?

The locker problem answer key is a fascinating mathematical puzzle that explores the concepts of probability, combinatorics, and systematic reasoning. This problem has captivated students, educators, and math enthusiasts alike due to its simplicity and the surprising results it yields. In this article, we will delve into the details of the locker problem, its variations, and the methodology used to arrive at the answer key.

Understanding the Locker Problem

The classic version of the locker problem can be articulated as follows:

- There are 1000 lockers in a row, all initially closed.

- 1000 students will sequentially go through the lockers.
- The first student will open every locker.
- The second student will toggle (close if open, open if closed) every second locker (i.e., lockers 2, 4, 6, ...).
- The third student will toggle every third locker (i.e., lockers 3, 6, 9, ...).
- This process continues until the 1000th student toggles the 1000th locker.

The challenge is to determine which lockers remain open after all students have completed their turns.

Analyzing the Problem

At first glance, the problem seems straightforward, but the underlying mechanics reveal a more complex pattern. The key to solving the locker problem lies in understanding how many times each locker is toggled throughout the process.

Counting Toggles

Each locker number corresponds to the number of students that toggle it. A locker will be toggled by a student if the student’s number is a divisor of the locker number. For instance:

- Locker 12 will be toggled by students 1, 2, 3, 4, 6, and 12 since these numbers are divisors of 12.

In general, a locker \(n \) will be toggled as many times as it has divisors. The critical insight here is that most numbers have an even number of divisors because they can be paired: for example, for locker 12, the pairs are (1, 12), (2, 6), and (3, 4).

However, perfect squares are unique because they have an odd number of divisors. This is because one of the divisors is repeated (e.g., for locker 36, the divisors are 1, 2, 3, 4, 6, 9, 12, 18, and 36, with the pair (6, 6) counted only once).

Identifying Open Lockers

Given the above reasoning, we can conclude that:

- Lockers that are toggled an odd number of times remain open.
- Therefore, only lockers that are perfect squares will remain open because they are the only ones with an odd number of divisors.

The perfect squares between 1 and 1000 are:

- 1 (1^2)
- 4 (2^2)
- 9 (3^2)
- 16 (4^2)

- 25 (5^2)
- 36 (6^2)
- 49 (7^2)
- 64 (8^2)
- 81 (9^2)
- 100 (10^2)
- 121 (11^2)
- 144 (12^2)
- 169 (13^2)
- 196 (14^2)
- 225 (15^2)
- 256 (16^2)
- 289 (17^2)
- 324 (18^2)
- 361 (19^2)
- 400 (20^2)
- 441 (21^2)
- 484 (22^2)
- 529 (23^2)
- 576 (24^2)
- 625 (25^2)
- 676 (26^2)
- 729 (27^2)
- 784 (28^2)
- 841 (29^2)
- 900 (30^2)
- 961 (31^2)

Thus, the total number of lockers that remain open after all students have toggled the lockers is 31.

Variations of the Problem

The locker problem can be modified in various ways to explore different mathematical concepts. Here are a few notable variations:

1. Different Numbers of Lockers and Students

What happens if the number of lockers and students is not the same? For example, if there are 500 lockers and 1000 students, the same principles apply. The lockers that remain open will still be the perfect squares, but the total number of open lockers will be limited to the perfect squares less than or equal to 500.

2. Changing the Toggling Pattern

If we change the pattern of toggling—say, if each student toggles lockers in a different sequence—the problem can yield different results. For example, if each student toggles every locker that is a multiple of their student number, the logic of divisors still applies, but the interaction becomes more complex.

3. Introducing Additional Rules

Adding rules, such as students closing lockers instead of toggling them, or allowing students to open lockers only if they are closed, can shift the problem's focus and lead to alternate conclusions.

Conclusion

The locker problem is not just a simple exercise in toggling lockers; it serves as a gateway into deeper mathematical concepts such as divisors, perfect squares, and systematic problem-solving. The solution reveals that only the perfect square lockers remain open after all students have completed their turns, leading to a total of 31 open lockers.

Whether approached as a classroom activity, a puzzle for math enthusiasts, or a topic of academic inquiry, the locker problem and its variations continue to engage minds and inspire curiosity about the elegance of mathematics. Understanding the locker problem enhances our appreciation for the interplay of numbers and the surprising patterns that emerge from seemingly simple rules.

Frequently Asked Questions

What is the locker problem in mathematics?

The locker problem is a classic problem that involves a number of lockers and a number of students. Each student toggles the state of the lockers (open or closed) based on specific rules, and the challenge is to determine which lockers remain open after all students have toggled them.

How many lockers are involved in the standard version of the locker problem?

In the standard version of the locker problem, there are typically 100 lockers, numbered from 1 to 100.

What is the rule for how students toggle the lockers?

The first student opens all the lockers. The second student toggles every second locker (locks 2, 4, 6, ...), the third student toggles every third locker (locks 3, 6, 9, ...), and this continues until the last student toggles the 100th locker.

What pattern emerges from the locker problem?

The pattern that emerges is that a locker will be open if it is toggled an odd number of times, which happens for lockers that are perfect squares (1, 4, 9, 16, ..., 100) because they have an odd number of divisors.

Which lockers remain open after all students have finished toggling?

The lockers that remain open are those numbered with perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100.

Why do only perfect square lockers remain open?

Only perfect square lockers remain open because they are the only ones with an odd number of factors, resulting in them being toggled an odd number of times.

Can the locker problem be generalized to any number of lockers?

Yes, the locker problem can be generalized to any number of lockers, and the result will be that only lockers corresponding to perfect squares up to that number will remain open.

What is the mathematical significance of the locker problem?

The locker problem illustrates concepts of divisors, parity (odd/even), and is often used in combinatorics and number theory, making it a popular problem in mathematical education.

Are there any real-world applications of the locker problem?

While primarily a theoretical problem, the principles behind it can be applied in computer science, particularly in understanding toggling, binary states, and optimization problems.

Where can I find the answer key for the locker problem?

The answer key for the locker problem can typically be found in math textbooks, educational websites, or specific problem-solving forums that focus on combinatorial problems.

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