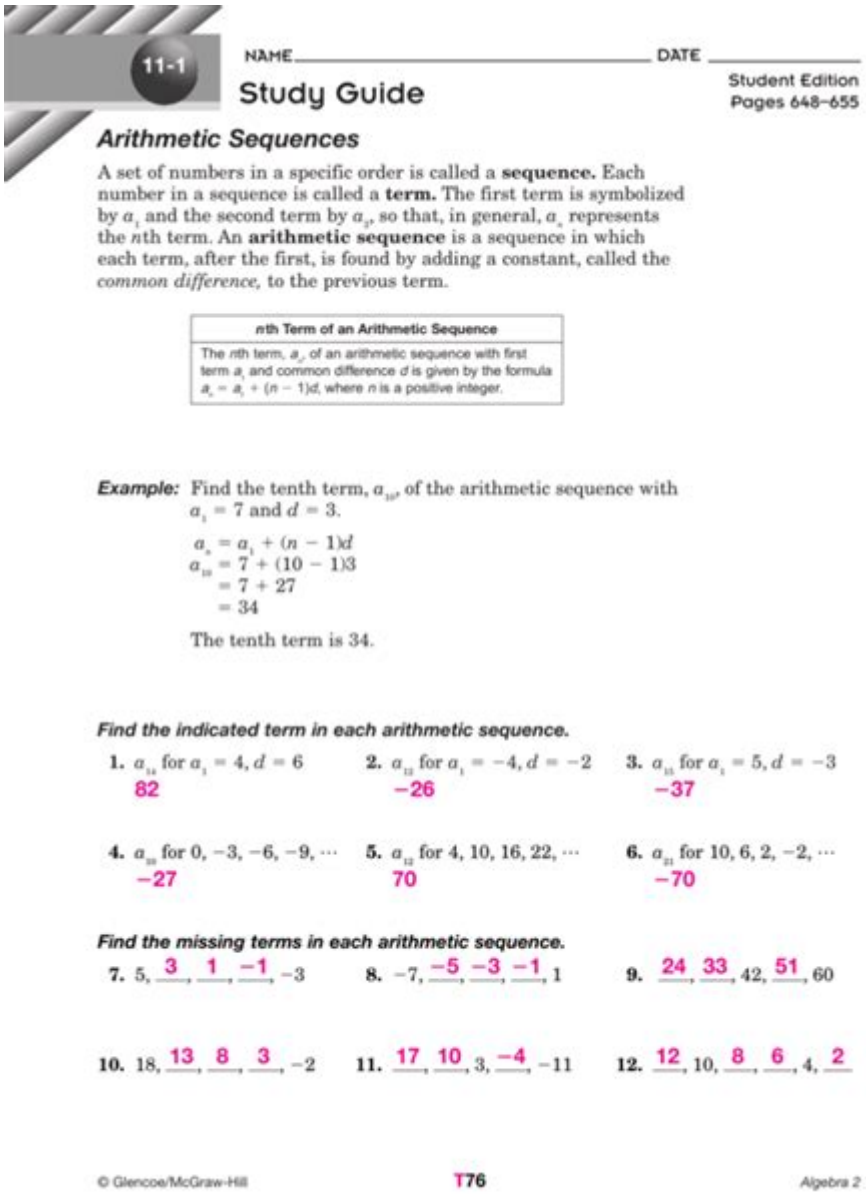
Study Guide For Arithmetic And Geometric Sequences



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Study Guide

Student Edition
Pages 648–655

Arithmetic Sequences

A set of numbers in a specific order is called a **sequence.** Each number in a sequence is called a **term.** The first term is symbolized by a_1 and the second term by a_2, so that, in general, a_n represents the nth term. An **arithmetic sequence** is a sequence in which each term, after the first, is found by adding a constant, called the *common difference,* to the previous term.

*n*th Term of an Arithmetic Sequence
The nth term, a_n, of an arithmetic sequence with first term a_1 and common difference d is given by the formula $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Example: Find the tenth term, a_{10}, of the arithmetic sequence with $a_1 = 7$ and $d = 3$.

$a_n = a_1 + (n - 1)d$
$a_{10} = 7 + (10 - 1)3$
$= 7 + 27$
$= 34$

The tenth term is 34.

Find the indicated term in each arithmetic sequence.

1. a_{14} for $a_1 = 4, d = 6$ — 82
2. a_{12} for $a_1 = -4, d = -2$ — −26
3. a_{15} for $a_1 = 5, d = -3$ — −37
4. a_{20} for $0, -3, -6, -9, \cdots$ — −27
5. a_{12} for $4, 10, 16, 22, \cdots$ — 70
6. a_{21} for $10, 6, 2, -2, \cdots$ — −70

Find the missing terms in each arithmetic sequence.

7. 5, 3, 1, −1, −3
8. −7, −5, −3, −1, 1
9. 24, 33, 42, 51, 60
10. 18, 13, 8, 3, −2
11. 17, 10, 3, −4, −11
12. 12, 10, 8, 6, 4, 2

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Study Guide for Arithmetic and Geometric Sequences

Understanding sequences is a fundamental concept in mathematics, especially in algebra and higher-level math. This study guide will delve into the intricacies of arithmetic and geometric sequences, exploring their definitions, formulas, applications, and how to identify and solve problems related to them. By the end of this guide, you will have a comprehensive understanding of these two types of sequences, enabling you to confidently tackle any related problem.

What is a Sequence?

A sequence is a list of numbers arranged in a specific order. Each number in a sequence is called a term. Sequences can be finite (having a limited number of terms) or infinite (continuing indefinitely). The two most common types of sequences are arithmetic and geometric sequences.

Arithmetic Sequences

An arithmetic sequence is a sequence of numbers in which the difference between consecutive terms is constant. This difference is known as the common difference (d).

Definition and Formula

- Definition: An arithmetic sequence can be defined as:
$$
a_n = a_1 + (n - 1)d
$$
where:
- a_n is the nth term,
- a_1 is the first term,
- d is the common difference, and
- n is the term number.

- Common Difference: The common difference can be found by subtracting any term from the subsequent term:
$$
d = a_{n} - a_{n-1}
$$

Examples of Arithmetic Sequences

1. Example 1: Consider the sequence 2, 5, 8, 11, 14.
- First term $a_1 = 2$
- Common difference $d = 5 - 2 = 3$

2. Example 2: The sequence 10, 7, 4, 1, -2.
- First term $a_1 = 10$

- Common difference $d = 7 - 10 = -3$

Finding Terms in an Arithmetic Sequence

To find any term in an arithmetic sequence, use the formula:

1. Identify the first term a_1 and the common difference d.
2. Plug in the values into the formula $a_n = a_1 + (n - 1)d$.

Example: Find the 10th term of the sequence 3, 7, 11, 15.

- Here, $a_1 = 3$ and $d = 4$.
- Using the formula:

$$
a_{10} = 3 + (10 - 1)4 = 3 + 36 = 39
$$

Sum of the First n Terms

The sum of the first n terms (S_n) of an arithmetic sequence can be calculated using the formula:

$$
S_n = \frac{n}{2} (a_1 + a_n)
$$

Alternatively, you can also use:

$$
S_n = \frac{n}{2} (2a_1 + (n - 1)d)
$$

Example: Find the sum of the first 5 terms of the sequence 1, 4, 7, 10, 13.

- Here, $n = 5$, $a_1 = 1$, and $a_5 = 13$.
- Using the first formula:

$$
S_5 = \frac{5}{2} (1 + 13) = \frac{5}{2} \times 14 = 35
$$

Geometric Sequences

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous term by a fixed, non-zero number called the common ratio (r).

Definition and Formula

- Definition: A geometric sequence can be defined as:
$$
a_n = a_1 \cdot r^{(n - 1)}
$$
where:
- a_n is the nth term,
- a_1 is the first term,
- r is the common ratio, and
- n is the term number.

- Common Ratio: The common ratio can be found by dividing any term by the previous term:
$$
r = \frac{a_n}{a_{n-1}}
$$

Examples of Geometric Sequences

1. Example 1: Consider the sequence 3, 9, 27, 81.
- First term $a_1 = 3$
- Common ratio $r = \frac{9}{3} = 3$

2. Example 2: The sequence 16, 8, 4, 2.
- First term $a_1 = 16$
- Common ratio $r = \frac{8}{16} = \frac{1}{2}$

Finding Terms in a Geometric Sequence

To find any term in a geometric sequence, use the formula:

1. Identify the first term a_1 and the common ratio r.

2. Plug in the values into the formula $a_n = a_1 \cdot r^{(n - 1)}$.

Example: Find the 5th term of the sequence 2, 6, 18, 54.

- Here, $a_1 = 2$ and $r = 3$.
- Using the formula:
$$
a_5 = 2 \cdot 3^{(5 - 1)} = 2 \cdot 81 = 162
$$

Sum of the First n Terms

The sum of the first n terms (S_n) of a geometric sequence can be calculated using the formula:
$$
S_n = a_1 \cdot \frac{1 - r^n}{1 - r} \quad \text{(for } r \neq 1\text{)}
$$

Example: Find the sum of the first 4 terms of the sequence 2, 6, 18, 54.

- Here, $a_1 = 2$, $r = 3$, and $n = 4$.
- Using the formula:
$$
S_4 = 2 \cdot \frac{1 - 3^4}{1 - 3} = 2 \cdot \frac{1 - 81}{-2} = 2 \cdot 40 = 80
$$

Applications of Sequences

Sequences find applications in various fields, including:

- Finance: Calculating compound interest, where funds increase geometrically.
- Computer Science: Algorithms often use sequences for efficiency.
- Physics: Describing patterns in motion or waves.
- Biology: Modeling population growth or decay.

Conclusion

This study guide has provided a thorough overview of arithmetic and geometric sequences, detailing their definitions, formulas, and applications. By mastering these concepts, you will enhance your problem-

solving skills and your understanding of mathematical principles. Remember to practice regularly and utilize these sequences in real-world scenarios to solidify your knowledge.

Frequently Asked Questions

What is the difference between an arithmetic sequence and a geometric sequence?

An arithmetic sequence is a sequence of numbers in which the difference between consecutive terms is constant, while a geometric sequence is a sequence where each term is found by multiplying the previous term by a fixed, non-zero number called the common ratio.

How do you find the nth term of an arithmetic sequence?

The nth term of an arithmetic sequence can be found using the formula: a_n = a_1 + (n - 1) d, where a_n is the nth term, a_1 is the first term, d is the common difference, and n is the term number.

What is the formula for the sum of the first n terms of an arithmetic sequence?

The formula for the sum of the first n terms of an arithmetic sequence is S_n = n/2 (2a_1 + (n - 1)d), where S_n is the sum of the first n terms, a_1 is the first term, d is the common difference, and n is the number of terms.

How do you calculate the nth term of a geometric sequence?

The nth term of a geometric sequence can be calculated using the formula: a_n = a_1 r^(n - 1), where a_n is the nth term, a_1 is the first term, r is the common ratio, and n is the term number.

What is the formula for the sum of the first n terms of a geometric sequence?

The formula for the sum of the first n terms of a geometric sequence is S_n = a_1 (1 - r^n) / (1 - r), where S_n is the sum of the first n terms, a_1 is the first term, r is the common ratio, and n is the number of terms, provided that r is not equal to 1.

Can you provide an example of how to identify whether a sequence is arithmetic or geometric?

To identify whether a sequence is arithmetic or geometric, examine the differences between consecutive terms. If they are constant, it's arithmetic. For geometric, check if each term is a constant multiple of the

previous one. For example, the sequence 2, 4, 6, 8 (constant difference of 2) is arithmetic, while 3, 6, 12, 24 (each term multiplied by 2) is geometric.

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