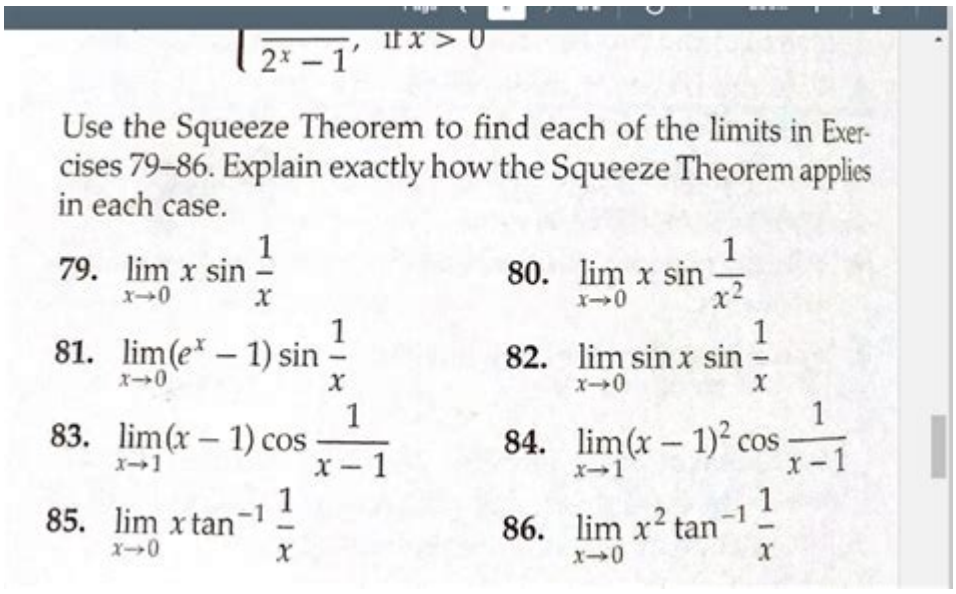
Squeeze Theorem Practice Problems



$\frac{}{2^x - 1}$, if $x > 0$

Use the Squeeze Theorem to find each of the limits in Exercises 79–86. Explain exactly how the Squeeze Theorem applies in each case.

79. $\lim_{x \to 0} x \sin \frac{1}{x}$

80. $\lim_{x \to 0} x \sin \frac{1}{x^2}$

81. $\lim_{x \to 0} (e^x - 1) \sin \frac{1}{x}$

82. $\lim_{x \to 0} \sin x \sin \frac{1}{x}$

83. $\lim_{x \to 1} (x - 1) \cos \frac{1}{x - 1}$

84. $\lim_{x \to 1} (x - 1)^2 \cos \frac{1}{x - 1}$

85. $\lim_{x \to 0} x \tan^{-1} \frac{1}{x}$

86. $\lim_{x \to 0} x^2 \tan^{-1} \frac{1}{x}$

Squeeze theorem practice problems are essential for students learning calculus, particularly when dealing with limits. The squeeze theorem, also known as the sandwich theorem, provides a method to determine the limit of a function by "squeezing" it between two other functions whose limits are known and equal. This article will explore the squeeze theorem, provide examples of practice problems, and guide you through the process of applying the theorem effectively.

Understanding the Squeeze Theorem

The squeeze theorem can be formally stated as follows:

If $f(x) \leq g(x) \leq h(x)$ for all x in some interval around c (except possibly at c), and if

$$
\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,
$$

then

$$
\lim_{x \to c} g(x) = L.
$$

This theorem is particularly useful in scenarios where direct substitution is difficult or impossible.

Key Concepts

1. Functions: We typically have three functions: $f(x)$, $g(x)$, and

$h(x)$.
2. Limit: The limit L is the value that both $f(x)$ and $h(x)$ approach as x approaches c.
3. Interval: The interval around c should exclude c itself.
4. Inequality: The inequalities $f(x) \leq g(x) \leq h(x)$ must hold true within the specified interval.

Practice Problems

Now that we've established a foundational understanding of the squeeze theorem, let’s dive into some practice problems that can help reinforce your knowledge.

Problem 1: Basic Application

Evaluate the limit:

$$
\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right).
$$

Solution Steps:

1. Identify Functions: We know that $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$.
2. Multiply by x^2: Thus,

$$
-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2.
$$

3. Evaluate Limits: Now calculate the limits of the bounding functions:

$$
\lim_{x \to 0} -x^2 = 0,
$$

$$
\lim_{x \to 0} x^2 = 0.
$$

4. Apply Squeeze Theorem: By the squeeze theorem, we conclude:

$$
\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.
$$

Problem 2: Trigonometric Limits

Evaluate:

$$
\lim_{x \to 0} x \tan(x).
$$

\]

Solution Steps:

1. Identify Functions: We know that

\[
\tan(x) \geq x \quad \text{for } x > 0,
\]

and

\[
\tan(x) \leq \frac{1}{\cos(x)} \quad \text{for all } x.
\]

Thus,

\[
x^2 \leq x \tan(x) \leq \frac{x}{\cos(x)}.
\]

2. Evaluate Limits: Evaluate the limits of the bounding functions:

\[
\lim_{x \to 0} x^2 = 0,
\]

\[
\lim_{x \to 0} \frac{x}{\cos(x)} = 0.
\]

3. Apply Squeeze Theorem: Conclusively,

\[
\lim_{x \to 0} x \tan(x) = 0.
\]

Problem 3: More Complex Functions

Evaluate:

\[
\lim_{x \to 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{x}.
\]

Solution Steps:

1. Rewrite the Expression: This can be simplified to

\[
\lim_{x \to 0} x \sin\left(\frac{1}{x}\right).
\]

2. Bounding Functions: We know $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$.

Thus,

$$
-x \leq x \sin\left(\frac{1}{x}\right) \leq x.
$$

3. Evaluate Limits: The bounding functions yield:

$$
\lim_{x \to 0} -x = 0,
$$

$$
\lim_{x \to 0} x = 0.
$$

4. Apply Squeeze Theorem: Therefore,

$$
\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0.
$$

Problem 4: Exponential Functions

Evaluate:

$$
\lim_{x \to 0} x^2 e^{-1/x^2}.
$$

Solution Steps:

1. Bounding Functions: For $x > 0$,

$$
0 \leq e^{-1/x^2} \leq 1.
$$

Thus,

$$
0 \leq x^2 e^{-1/x^2} \leq x^2.
$$

2. Evaluate Limits:

$$
\lim_{x \to 0} 0 = 0,
$$

$$
\lim_{x \to 0} x^2 = 0.
$$

3. Apply Squeeze Theorem: Therefore,

$$
\lim

$$\lim_{x \to 0} x^2 e^{-1/x^2} = 0.$$

Practice Problem Summary

To effectively use the squeeze theorem, follow these steps:

1. Identify the Function: Determine the function for which you want to find the limit.
2. Find Bounding Functions: Establish two functions that "squeeze" your function.
3. Evaluate Limits: Calculate the limits of the bounding functions.
4. Apply the Theorem: Use the squeeze theorem to conclude the limit of your function.

Additional Practice Problems

Here are some additional problems for practice:

1. Evaluate $\lim_{x \to 0} \frac{\sin(5x)}{x}$.
2. Evaluate $\lim_{x \to 1} (x^2 - 1) \sin\left(\frac{1}{x-1}\right)$.
3. Evaluate $\lim_{x \to \infty} \frac{\sin(x)}{x}$.

Conclusion

The squeeze theorem is a powerful tool for evaluating limits, especially in situations where traditional methods may fail. Through practice problems, we gain insight into how to effectively apply the theorem. Mastering the concepts and techniques associated with the squeeze theorem will strengthen your calculus skills and prepare you for more advanced topics in mathematics. Always remember the three-step process of identifying functions, bounding them, and applying the theorem for a successful limit evaluation. Happy studying!

Frequently Asked Questions

What is the Squeeze Theorem?

The Squeeze Theorem states that if you have three functions, f(x), g(x), and h(x), and if $f(x) \leq g(x) \leq h(x)$ for all x in some interval around a point a (except possibly at a), and if the limits of f(x) and h(x) as x approaches a are both L, then the limit of g(x) as x approaches a is also L.

How do I apply the Squeeze Theorem to find limits?

To apply the Squeeze Theorem, identify two functions that bound the function of interest from below and above. Ensure their limits are equal at the point of interest, then conclude that the limit of the function in question must also equal that value.

Can you give an example of a Squeeze Theorem problem?

Sure! Consider the limit: lim (x -> 0) x^2 sin(1/x). We can use the fact that -1 ≤ sin(1/x) ≤ 1 to squeeze it: -x^2 ≤ x^2 sin(1/x) ≤ x^2. Taking limits, both -x^2 and x^2 approach 0 as x approaches 0, hence by the Squeeze Theorem, lim (x -> 0) x^2 sin(1/x) = 0.

What are common mistakes when using the Squeeze Theorem?

Common mistakes include failing to establish that the bounding functions converge to the same limit, not confirming that the inequalities hold for all x in the interval, or misapplying the theorem by assuming the middle function has a limit without proper justification.

How do I know when to use the Squeeze Theorem?

Use the Squeeze Theorem when you have a function that is difficult to evaluate directly, but can be bounded by simpler functions whose limits are easier to compute, especially when dealing with oscillating functions or indeterminate forms.

Is the Squeeze Theorem applicable for limits at infinity?

Yes, the Squeeze Theorem can be used for limits at infinity as well. Just ensure that you can find two functions that bound your function from above and below and that both of those functions converge to the same limit as x approaches infinity.

What is a real-world application of the Squeeze Theorem?

One real-world application of the Squeeze Theorem is in physics, particularly in analyzing the behavior of waves or oscillations where a function may be bounded by two other functions, allowing for predictions about the system's limits without direct calculation.

Squeeze Theorem Practice Problems

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matlab常用函数之squeeze函数 - 百度经验
May 30, 2015 · matlab中squeeze函数用于删除矩阵中的单一维（Remove singleton dimensions），比如执行下面的代码，随机产生一个1x2x3的矩阵A，然后squeeze（A）将返回一个2x3的矩阵，将第一维却掉 (因为第一位大小为1):

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Jun 22, 2010 · The origin of the word squeeze is obscure. The apparent similarity to freeze is what leads some to think squoze is correct. However, I've only ever heard it in a jocular vein, a weak attempt at humour. I would say "squeezed" is the correct answer

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Dec 1, 2020 · Hi, Is it idiomatic to say "Although he has a very busy schedule, he might be able to squeeze out (some) time for a personal meeting." when talking about a high official? The person speaking is his secretary. Thanks in advance!

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Oct 27, 2007 · Does anybody know the origins of the expression "the eagle shits today"? Yeah, I know, this is not my first question concerning idiomatic sayings featuring shit, this is purely accidental, I hasten to assure.

press / squeeze - WordReference Forums
Sep 20, 2017 · Hi, What is the difference between press and squeeze? I press /squeeze the index finger on my left hand with my right hand. I press /squeeze the orange to get the juice. I press /squeeze the dough. Thank you very much.

Someone's squeeze - WordReference Forums
Feb 3, 2010 · Hello, I am trying to make out this sentence : "You may be Harvey's Squeeze" Context : Harvey is a District Attorney. And the person who said that sentence is the first time he sees the woman that he calls " Harvey's squeeze" Anyone for an idea ? Does "someone's squeeze" has a sense more...

The juice is worth the squeeze... | WordReference Forums
Oct 20, 2010 · I believe this phrase is used mostly with negative verb: " The juice ain't worth the squeeze. " In this case the adequate translation would be : " Овчинка выделки не стоит ". (literally

"the sheepskin ain't worth the dressing".) However, if you need the positive usage, "the juice is worth the squeeze", then you could say " дело того стоит " (the business is ...

Squeeze the trigger/Pull the trigger - WordReference Forums
Jun 22, 2009 · How to describe an action of shooting by a gun? Squeeze the trigger or Pull the trigger? My personal thinking, this action is "to squeeze" rather than "to pull". Which is correct? If both of them are correct, is there any difference between those 2 idioms?

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