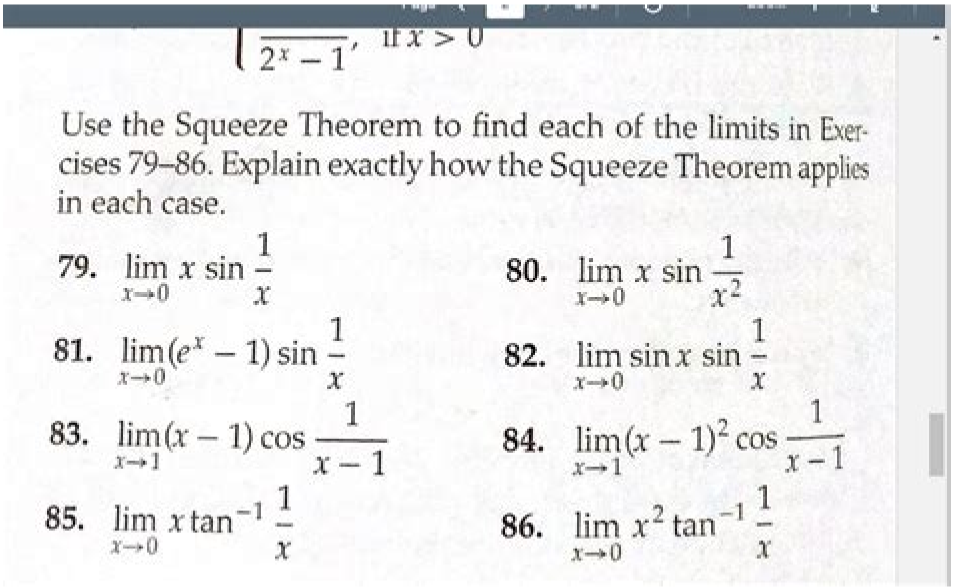
# Squeeze Theorem Questions And Answers



$\frac{\ldots}{2^x - 1}$, if $x > 0$

Use the Squeeze Theorem to find each of the limits in Exercises 79–86. Explain exactly how the Squeeze Theorem applies in each case.

79. $\lim_{x \to 0} x \sin \frac{1}{x}$

80. $\lim_{x \to 0} x \sin \frac{1}{x^2}$

81. $\lim_{x \to 0} (e^x - 1) \sin \frac{1}{x}$

82. $\lim_{x \to 0} \sin x \sin \frac{1}{x}$

83. $\lim_{x \to 1} (x - 1) \cos \frac{1}{x - 1}$

84. $\lim_{x \to 1} (x - 1)^2 \cos \frac{1}{x - 1}$

85. $\lim_{x \to 0} x \tan^{-1} \frac{1}{x}$

86. $\lim_{x \to 0} x^2 \tan^{-1} \frac{1}{x}$

**Squeeze theorem questions and answers** are vital for understanding limits in calculus. The Squeeze Theorem, also known as the Sandwich Theorem, is a powerful tool for determining the limits of functions that are difficult to evaluate directly. This article will delve into the Squeeze Theorem, explore various types of questions related to it, and provide comprehensive answers to enhance your understanding.

## Understanding the Squeeze Theorem

The Squeeze Theorem states that if you have three functions, $f(x)$, $g(x)$, and $h(x)$, defined on an interval around $a$, and if the following conditions are satisfied:

1. $f(x) \leq g(x) \leq h(x)$ for all $x$ in that interval (except possibly at $x = a$),
2. $\lim_{x \to a} f(x) = L$,
3. $\lim_{x \to a} h(x) = L$,

then it follows that:

$$\lim_{x \to a} g(x) = L$$

This theorem is especially useful when $g(x)$ is challenging to evaluate directly, but $f(x)$ and $h(x)$ are simpler functions whose limits are known.

# Common Questions about the Squeeze Theorem

In this section, we will outline some common questions related to the Squeeze Theorem, including examples and their solutions.

## 1. Basic Application of the Squeeze Theorem

Question: Evaluate the limit:

$$
\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right).
$$

Answer: To apply the Squeeze Theorem, we need to find two functions that bound $g(x) = x^2 \sin\left(\frac{1}{x}\right)$.

Since $\sin(t)$ oscillates between -1 and 1, we can write:

$$
-1 \leq \sin\left(\frac{1}{x}\right) \leq 1.
$$

Multiplying through by $x^2$ (which is non-negative for $x$ near 0), we get:

$$
-x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2.
$$

Now, we can evaluate the limits of the bounding functions as $x$ approaches 0:

$$
\lim_{x \to 0} -x^2 = 0 \quad \text{and} \quad \lim_{x \to 0} x^2 = 0.
$$

By the Squeeze Theorem:

$$
\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.
$$

## 2. Squeeze Theorem with Polynomial Functions

Question: Prove that:

$$
\lim_{x \to 1} (x^2 + x - 2) = 0
$$

\]

using the Squeeze Theorem.

Answer: We first need to factor the polynomial:

\[
x^2 + x - 2 = (x - 1)(x + 2).
\]

When \(x\) is near 1, we can bound this function. Notice that:

- For \(x\) close to 1, \(x - 1\) is small.
- The value of \(x + 2\) is approximately 3 when \(x = 1\).

Hence, we can find bounds:

\[
-3 |x - 1| \leq (x - 1)(x + 2) \leq 3 |x - 1|.
\]

As \(x\) approaches 1, both bounds approach 0:

\[
\lim_{x \to 1} -3 |x - 1| = 0 \quad \text{and} \quad \lim_{x \to 1} 3 |x - 1| = 0.
\]

By the Squeeze Theorem, we have:

\[
\lim_{x \to 1} (x^2 + x - 2) = 0.
\]

## 3. Trigonometric Functions and the Squeeze Theorem

Question: Show that:

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1
\]

using the Squeeze Theorem.

Answer: Consider the unit circle. For \(0 < x < \frac{\pi}{2}\):

\[
\sin x < x < \tan x.
\]

Dividing all parts by $\sin x$ (which is positive near 0), we get:

$$
1 < \frac{x}{\sin x} < \frac{1}{\cos x}.
$$

Taking the reciprocal (and reversing the inequalities):

$$
\cos x < \frac{\sin x}{x} < 1.
$$

As $x$ approaches 0, we know:

$$
\lim_{x \to 0} \cos x = 1.
$$

Thus, by the Squeeze Theorem:

$$
\lim_{x \to 0} \frac{\sin x}{x} = 1.
$$

## 4. Squeeze Theorem in Complex Limits

Question: Evaluate the limit:

$$
\lim_{x \to \infty} \frac{\sin^2 x}{x^2}.
$$

Answer: To apply the Squeeze Theorem, we note that $\sin^2 x$ oscillates between 0 and 1, so we can state:

$$
0 \leq \sin^2 x \leq 1.
$$

Dividing through by $x^2$ gives:

$$
0 \leq \frac{\sin^2 x}{x^2} \leq \frac{1}{x^2}.
$$

As $x$ approaches infinity:

$$
\lim_{x \to \infty} 0 = 0 \quad \text{and} \quad \lim_{x \to \infty} \frac{1}{x^2} = 0.

\]

By the Squeeze Theorem:

$$
\lim_{x \to \infty} \frac{\sin^2 x}{x^2} = 0.
$$

# Conclusion

The Squeeze Theorem is an essential concept in calculus, particularly useful for evaluating limits of complex functions. By understanding its application through various examples, students can better grasp the behavior of functions at specific points. The questions and answers provided in this article illustrate not only the utility of the theorem but also its adaptability across different types of functions, including polynomials, trigonometric functions, and others. Mastery of the Squeeze Theorem will undoubtedly enhance your calculus skills and problem-solving abilities.

# Frequently Asked Questions

## What is the Squeeze Theorem in calculus?

The Squeeze Theorem states that if you have three functions, f(x), g(x), and h(x), and if $f(x) \leq g(x) \leq h(x)$ for all x in an interval around a point 'a', and if the limits of f(x) and h(x) as x approaches 'a' are both equal to L, then the limit of g(x) as x approaches 'a' is also L.

## How do you apply the Squeeze Theorem to find the limit of sin(x)/x as x approaches 0?

To apply the Squeeze Theorem, note that for x near 0, sin(x) is bounded by -1 and 1. Therefore, $0 \leq \sin(x) \leq x$ for $x > 0$ and $-x \leq \sin(x) \leq 0$ for $x < 0$. This implies that $0 \leq \sin(x)/x \leq 1$ for $x > 0$, and by taking limits, we find that both the upper and lower bounds approach 1 as x approaches 0.

## Can the Squeeze Theorem be used for sequences as well as functions?

Yes, the Squeeze Theorem can also be applied to sequences. If you have three sequences $a_n$, $b_n$, and $c_n$ such that $a_n \leq b_n \leq c_n$ for all n, and if the limits of $a_n$ and $c_n$ as n approaches infinity are both equal to L, then the limit of $b_n$ as n approaches infinity is also L.

## What is a common mistake when using the Squeeze Theorem?

A common mistake is to incorrectly assume that if $f(x) \leq g(x) \leq h(x)$, then the limits of f(x) and h(x) approaching a certain value automatically guarantees that g(x) will also approach that value. It is essential that the limits of f(x) and h(x) converge to the same limit for the theorem to be valid.

## Give an example of a function where the Squeeze Theorem is applicable.

Consider the function $g(x) = x^2 \sin(1/x)$ as x approaches 0. We can squeeze it between $-x^2$ and $x^2$ since $-1 \leq \sin(1/x) \leq 1$. Hence, $-x^2 \leq g(x) \leq x^2$. As x approaches 0, both $-x^2$ and $x^2$ approach 0. Therefore, by the Squeeze Theorem, the limit of g(x) as x approaches 0 is also 0.

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