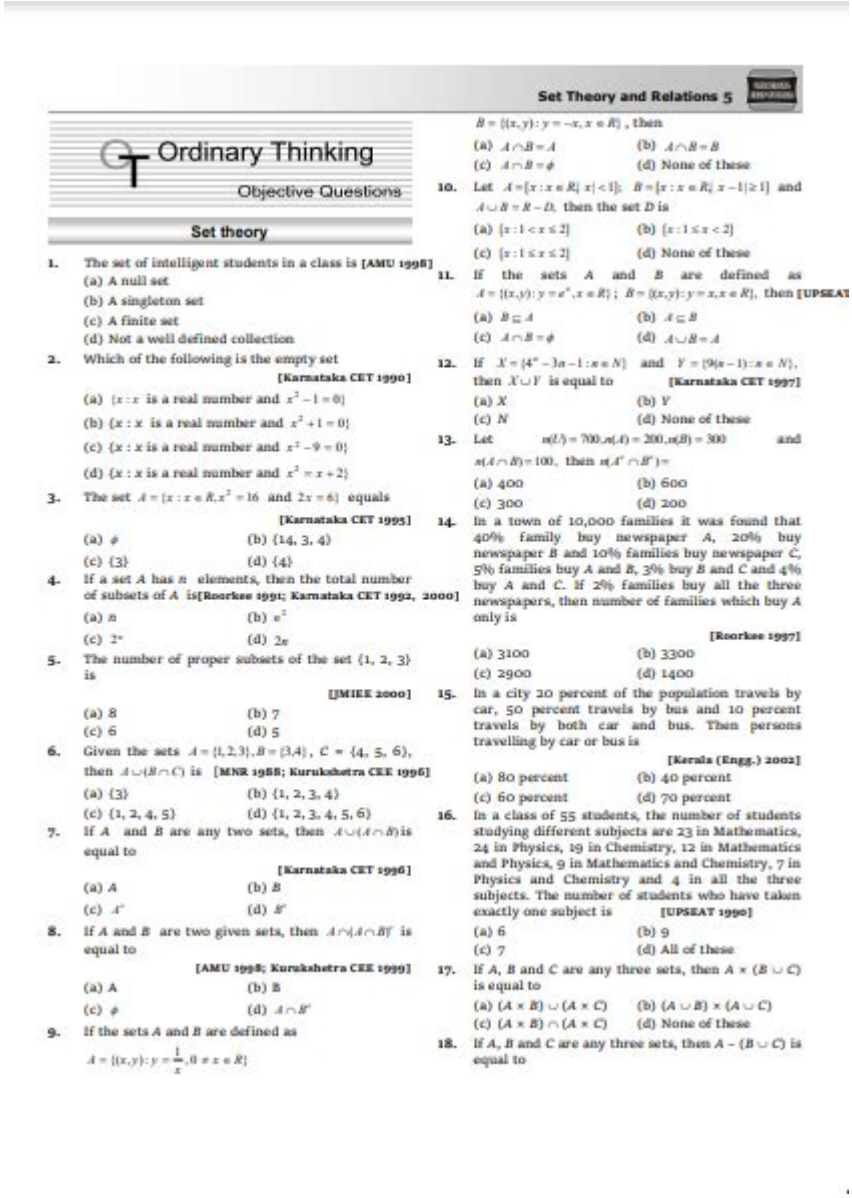
Set Theory Practice Problems



Set Theory and Relations 5

Ordinary Thinking

Objective Questions

Set theory

1. The set of intelligent students in a class is **[AMU 1998]**
   (a) A null set
   (b) A singleton set
   (c) A finite set
   (d) Not a well defined collection
2. Which of the following is the empty set **[Karnataka CET 1990]**
   (a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$
   (b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$
   (c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$
   (d) $\{x : x$ is a real number and $x^2 = x + 2\}$
3. The set $A = \{x : x \in R, x^2 = 16$ and $2x = 6\}$ equals **[Karnataka CET 1995]**
   (a) ϕ (b) {14, 3, 4}
   (c) {3} (d) {4}
4. If a set A has n elements, then the total number of subsets of A is **[Roorkee 1991; Karnataka CET 1992, 2000]**
   (a) n (b) n^2
   (c) 2^n (d) $2n$
5. The number of proper subsets of the set {1, 2, 3} is **[JMIEE 2000]**
   (a) 8 (b) 7
   (c) 6 (d) 5
6. Given the sets $A = \{1, 2, 3\}, B = \{3, 4\}, C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is **[MNR 1988; Kurukshetra CEE 1996]**
   (a) {3} (b) {1, 2, 3, 4}
   (c) {1, 2, 4, 5} (d) {1, 2, 3, 4, 5, 6}
7. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to **[Karnataka CET 1996]**
   (a) A (b) B
   (c) A' (d) B'
8. If A and B are two given sets, then $A \cap (A \cap B)'$ is equal to **[AMU 1998; Kurukshetra CEE 1999]**
   (a) A (b) B
   (c) ϕ (d) $A \cap B'$
9. If the sets A and B are defined as $A = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$, $B = \{(x, y) : y = -x, x \in R\}$, then
   (a) $A \cap B = A$ (b) $A \cap B = B$
   (c) $A \cap B = \phi$ (d) None of these
10. Let $A = \{x : x \in R, |x| < 1\}$; $B = \{x : x \in R, |x - 1| \geq 1\}$ and $A \cup B = R - D$, then the set D is
    (a) $\{x : 1 < x \leq 2\}$ (b) $\{x : 1 \leq x < 2\}$
    (c) $\{x : 1 \leq x \leq 2\}$ (d) None of these
11. If the sets A and B are defined as $A = \{(x, y) : y = e^x, x \in R\}$; $B = \{(x, y) : y = x, x \in R\}$, then **[UPSEAT**
    (a) $B \subseteq A$ (b) $A \subseteq B$
    (c) $A \cap B = \phi$ (d) $A \cup B = A$
12. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, then $X \cup Y$ is equal to **[Karnataka CET 1997]**
    (a) X (b) Y
    (c) N (d) None of these
13. Let $n(U) = 700, n(A) = 200, n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B') =$
    (a) 400 (b) 600
    (c) 300 (d) 200
14. In a town of 10,000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, then number of families which buy A only is **[Roorkee 1997]**
    (a) 3100 (b) 3300
    (c) 2900 (d) 1400
15. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is **[Kerala (Engg.) 2002]**
    (a) 80 percent (b) 40 percent
    (c) 60 percent (d) 70 percent
16. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is **[UPSEAT 1990]**
    (a) 6 (b) 9
    (c) 7 (d) All of these
17. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to
    (a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$
    (c) $(A \times B) \cap (A \times C)$ (d) None of these
18. If A, B and C are any three sets, then $A - (B \cup C)$ is equal to

Set theory practice problems are an essential part of understanding the foundational concepts of mathematics. Set theory, which deals with the study of sets—collections of objects—forms the basis for various areas of mathematics, including logic, probability, and statistics. This article will provide a comprehensive overview of set theory, introduce various types of problems, and offer practice exercises to enhance your understanding.

Understanding Set Theory Basics

Set theory originated in the late 19th century and has since become a fundamental theory in mathematics. Here are some key concepts:

Basic Definitions

1. Set: A collection of distinct objects, considered as a whole. Sets are usually denoted by capital letters (e.g., A, B, C).
2. Element: An object in a set. If an element a is in a set A, we write $a \in A$.
3. Empty Set: A set with no elements, denoted by $\emptyset$ or { }.
4. Subset: A set A is a subset of a set B if every element of A is also in B, denoted $A \subseteq B$.
5. Universal Set: The set that contains all possible elements under consideration, often denoted by U.
6. Intersection: The set of elements common to both sets A and B, denoted $A \cap B$.
7. Union: The set of all elements in either set A or set B, denoted $A \cup B$.
8. Complement: The set of all elements in the universal set U that are not in set A, denoted A' or $\overline{A}$.

Types of Sets

- Finite Set: A set with a limited number of elements (e.g., {1, 2, 3}).
- Infinite Set: A set with an unlimited number of elements (e.g., the set of natural numbers, $\mathbb{N}$).
- Countable Set: A set that can be put into one-to-one correspondence with the natural numbers.
- Uncountable Set: A set that cannot be counted, such as the set of real numbers.

Set Notation and Venn Diagrams

Understanding set notation and how to represent sets visually is crucial for solving set theory problems.

Set Notation

Set notation provides a concise way to describe sets. Here are a few notations:

- Roster or Tabular Form: Listing all elements (e.g., $A = \{1, 2, 3\}$).
- Set-builder Notation: Describes the properties that its members must satisfy (e.g., $B = \{x \mid x \text{ is an even number}\}$).

Venn Diagrams

Venn diagrams are visual tools used to represent sets and their relationships. A Venn diagram consists of circles that represent different sets, with overlapping areas showing intersections. They help in understanding unions, intersections, and complements visually.

Practice Problems in Set Theory

Now that we have a grasp of the basics, let's dive into some practice problems. These problems will range from basic to more complex concepts.

Problem Set 1: Basic Operations

1. Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$.
- a) Find $A \cap B$.
- b) Find $A \cup B$.
- c) Find A' if the universal set $U = \{1, 2, 3, 4, 5, 6\}$.

2. Given sets $C = \{x \mid x \text{ is a prime number less than } 10\}$ and $D = \{2, 3, 5, 7\}$:
- a) Determine if $C = D$.
- b) List the elements of C.

Problem Set 2: Subsets and Complements

1. If $E = \{2, 4, 6\}$, determine the subsets of E.
2. For the universal set $U = \{1, 2, 3, 4, 5\}$ and set $F = \{1, 3\}$:
- a) Find the complement F'.
- b) How many subsets does F have?

Problem Set 3: Word Problems

1. In a class of 30 students, 18 students play football, and 12 students play basketball. If 6 students play both sports, how many students play either football or basketball?
2. A survey of 100 people showed that 60 like coffee, 45 like tea, and 25 like both. How many people do not like either coffee or tea?

Solutions to Practice Problems

Here we will provide solutions to the problems outlined above.

Solutions to Problem Set 1

1.
- a) $A \cap B = \{3, 4\}$
- b) $A \cup B = \{1, 2, 3, 4, 5, 6\}$
- c) $A' = \{5, 6\}$

2.
- a) $C = D$ is true, since both contain the same elements.
- b) The elements of C are $\{2, 3, 5, 7\}$.

Solutions to Problem Set 2

1. The subsets of $E = \{2, 4, 6\}$ are:
- $\emptyset, \{2\}, \{4\}, \{6\}, \{2, 4\}, \{2, 6\}, \{4, 6\}, \{2, 4, 6\}$

2.
- a) $F' = \{2, 4, 5\}$
- b) F has $2^2 = 4$ subsets.

Solutions to Problem Set 3

1.
- Total students who play either sport = Students who play football + Students who play basketball - Students who play both = $18 + 12 - 6 = 24$.
- Thus, 6 students do not play either sport (30 - 24).

2.
- Total who do not like either = Total surveyed - (Those who like coffee + Those who like tea - Those who like both) = $100 - (60 + 45 - 25) = 100 - 80 = 20$.

Conclusion

Set theory forms the backbone of modern mathematics and is critical to understanding relationships between different mathematical concepts. By engaging with practice problems, learners can reinforce their understanding and application of these foundational principles. The practice problems outlined in this article cover basic operations, subsets, and real-world applications of set theory, providing a well-rounded approach to mastering this essential mathematical discipline. As you continue to explore set theory, remember that practice is key to deepening your understanding and proficiency in this fascinating area of mathematics.

Frequently Asked Questions

What is the union of two sets and how is it represented in set theory?

The union of two sets A and B, denoted as $A \cup B$, is the set containing all elements that are in A, in B, or in both. For example, if $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cup B = \{1, 2, 3, 4, 5\}$.

How do you find the intersection of two sets?

The intersection of two sets A and B, denoted as $A \cap B$, is the set containing all elements that are common to both sets. For instance, if $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A \cap B = \{2, 3\}$.

What are the differences between subsets and proper subsets in set theory?

A set A is a subset of set B (denoted $A \subseteq B$) if all elements of A are also in B. A proper subset (denoted $A \subset B$) is a subset that contains at least one element not in B, meaning A cannot be equal to B. For example, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$, then $A \subset B$.

How can Venn diagrams be used to solve set theory problems?

Venn diagrams visually represent sets and their relationships. By drawing circles for each set and shading areas for unions, intersections, or differences, students can easily visualize and solve problems related to set operations and their relationships.

What is the complement of a set and how is it determined?

The complement of a set A, denoted as A', includes all elements in the universal set U that are not in A. To find A', you identify all elements in U and exclude those in A. For example, if $U = \{1, 2, 3, 4, 5\}$ and $A = \{2, 3\}$, then $A' = \{1, 4, 5\}$.

Set Theory Practice Problems

C++ STL set详解-CSDN博客
Sep 20, 2016 · set作为一个容器也是用来存储同一数据类型的数据类型，并且能从一个数据集合中取出数据，在set中每个元素的值都唯一，而且系统能根据元素的值自动进行排序。

Python set () 函数 - 菜鸟教程
Python set () 函数 Python 内置函数 描述 set () 函数创建一个无序不重复元素集，可进行关系测试，删除重复数据，还可以计算交集、差集、并集等。

[C++ STL] set使用详解 - fengMisaka - 博客园
Jan 12, 2019 · 一、概述 set 容器内的元素会被自动排序，set 与 map 不同，set 中的元素即是键值又是实值，set 不允许两个元素有相同的键值。

C++ STL set容器完全攻略（超级详细） - C语言中文网
前面章节讲解了 map 容器和 multimap 容器的用法，类似地， C++ STL 标准库中还提供有 set 和 multiset 这 2 个容器，它们也属于关联式容器。不过，本节先讲解 set 容器，后续章节再讲解 …

std::set - C++中文 - API参考文档
std::set 是关联容器，含有 Key 类型对象的已排序集。用比较函数 比较 (Compare) 进行排序。搜索、移除和插入拥有对数复杂度。 set 通常以 红黑树 实现。在每个标准库使用 比较 …

C++ 容器类 | 菜鸟教程

C++ 标准库中的 是一个关联容器，它存储了一组唯一的元素，并按照一定的顺序进行排序。提供了高效的元素查找、插入和删除操作。

【总结】**C++ 基础数据结构 —— STL之集合（set）用法详解_std::set ...**

Oct 5, 2019 · 本文深入解析C++ STL中的set容器，涵盖概念、基本操作与应用实例，如插入、查找、删除等，并探讨自定义比较函数的实现。

C++之STL整理（4）之set 用法（创建、赋值、增删查改）详解_c++ set ...

Mar 29, 2024 · 一、set的初始化 STL 中的 set 是一个关联容器，它存储的元素都是 键值一体的（key-value pair），并且也会 根据键（key）自动排序。

SET中文 (简体)翻译：剑桥词典 - Cambridge Dictionary

If a story, film, etc. is set in a particular time or place, the action in it happens in that time or place.

《Set》Java中的Set用法（详细整理） - CSDN博客

Nov 18, 2020 · 本文介绍了Java中常见的Set类型，包括HashSet和TreeSet。HashSet元素唯一、无序，可含空元素；TreeSet元素唯一、有序，不能含空元素。

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Python set () 函数 - 菜鸟教程

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