Related Rates Worksheet

Homework 2.6 Related Rates day 2

2005 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

5. Consider the curve given by $y^2 = 2 + xy$.

   (a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

   (b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

   (c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

   (d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

2004 AP Calculus Free-Response Question

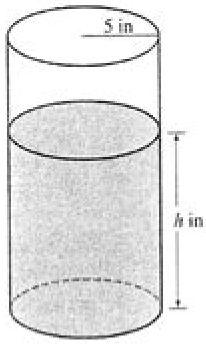
4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

   (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

   (b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

   (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.

2003 AP Calculus Free-Response Question



5. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

   (a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

   (b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

   (c) At what time t is the coffeepot empty?

Related rates worksheet is an essential tool in calculus that helps students and educators alike understand the concept of related rates in a practical and application-oriented manner. In calculus, related rates problems involve finding the rate at which one quantity changes concerning another, often in real-world scenarios. This article will explore the concept of related rates, provide examples, and offer a structured worksheet that educators can use to enhance students' understanding of the topic.

Understanding Related Rates

Related rates problems typically involve two or more variables that are related to each other through a specific equation. When one of these variables changes over time, it can affect the rate of change of the other variable. The primary goal in these problems is to find the unknown rate of change by applying differentiation, specifically implicit differentiation, to the relationship between the variables.

Key Concepts

1. Variables: Identify all the variables involved in the problem. These can be dimensions, angles, or other quantities that change over time.

2. Relationships: Establish the relationship between the variables using an equation. This equation is often derived from geometric or physical principles.

3. Rates: Determine the rates of change for the known variables. This is often given in the problem statement.

4. Differentiation: Use implicit differentiation to find the rate of change of the unknown variable concerning time.

5. Substitution: Substitute known values into the differentiated equation to solve for the unknown rate.

Common Examples of Related Rates Problems

To better comprehend related rates, let’s look at a few classic examples that illustrate how to approach these problems.

Example 1: The Balloon Problem

Imagine a balloon that is being inflated, and its radius increases at a rate of 2 cm/min. We want to find the rate at which the volume of the balloon is increasing.

1. Identify Variables:
- Let r be the radius (in cm).
- Let V be the volume of the balloon (in cm^3).

2. Relationship:
- The volume of a sphere is given by the formula $V = \frac{4}{3} \pi r^3$.

3. Known Rate:
- The radius increases at $\frac{dr}{dt} = 2$ cm/min.

4. Differentiate:
- Using implicit differentiation, we find:

$$
\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}
$$

5. Substitute Values:
- If $r = 5$ cm, then:

$$
\frac{dV}{dt} = 4\pi (5^2)(2) = 200\pi \text{ cm}^3/\text{min}
$$

Thus, the volume of the balloon is increasing at 200π cm^3/min when the radius is 5 cm.

Example 2: The Ladder Problem

Consider a ladder that is leaning against a wall. The base of the ladder is being pulled away from the wall at a rate of 1 ft/s. If the ladder is 10 ft long, how fast is the top of the ladder sliding down the wall when the base is 6 ft from the wall?

1. Identify Variables:
- Let x be the distance from the wall to the base of the ladder (in ft).
- Let y be the height of the top of the ladder (in ft).
- The length of the ladder is constant at $L = 10$ ft.

2. Relationship:
- By the Pythagorean theorem, we have:

$$
x^2 + y^2 = L^2 \Rightarrow x^2 + y^2 = 100
$$

3. Known Rate:
- The base of the ladder is moving away from the wall at $\frac{dx}{dt} = 1$ ft/s.

4. Differentiate:
- Differentiating both sides gives:

$$
2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0
$$

- Simplifying:

$$
y\frac{dy}{dt} = -x\frac{dx}{dt}
$$

5. Substitute Values:
- When $x = 6$:

$$
y^2 = 100 - 6^2 = 64 \Rightarrow y = 8
$$

- Now substitute:
$$
8\frac{dy}{dt} = -6(1) \Rightarrow \frac{dy}{dt} = -\frac{6}{8} = -\frac{3}{4} \text{ ft/s}
$$

Therefore, the top of the ladder is sliding down the wall at a rate of $\frac{3}{4}$ ft/s when the base is 6 ft from the wall.

Creating a Related Rates Worksheet

To facilitate learning and practice in related rates, educators can create a worksheet with various problems. Below is a suggested structure for a related rates worksheet.

Worksheet Structure

1. Introduction:
- Briefly explain what related rates are and how they are used in calculus.

2. Example Problems:
- Provide 2-3 solved examples similar to the ones discussed above.

3. Practice Problems:
- Create a list of problems for students to solve. Here are some suggested problems:

1. A conical tank is being filled with water. The radius of the base of the cone is increasing at a rate of 1 cm/s. How fast is the height of the water changing when the radius is 3 cm?

2. A car is 30 ft from a streetlight and is moving away from it at 5 ft/s. How fast is the angle of elevation from the tip of the shadow to the top of the streetlight changing when the car is 30 ft away?

3. A spherical snowball melts at a rate of 4 cm^3/min. How fast is the radius of the snowball decreasing when the radius is 3 cm?

4. Reflection Questions:
- Ask students to reflect on the methods used in solving related rates problems and how they can apply these concepts in real-life scenarios.

Conclusion

A **related rates worksheet** serves as an invaluable resource in teaching and learning calculus. By understanding the fundamental concepts of related rates and practicing through examples and exercises, students can develop a strong grasp of how different quantities are interconnected through rates of change. This understanding not only enhances their problem-solving skills in mathematics but also prepares them for real-world applications where these principles are relevant. Whether in physics, engineering, or everyday life, the ability to analyze and compute related rates is a critical skill for students to master.

Frequently Asked Questions

What are related rates in calculus?

Related rates are a method used in calculus to find the rate at which one quantity is changing in relation to another quantity that is also changing.

How do you set up a related rates problem?

To set up a related rates problem, identify the quantities that are changing, establish a relationship between them using an equation, and then differentiate with respect to time.

What is the importance of diagrams in related rates problems?

Diagrams help visualize the relationships between different quantities, making it easier to set up equations and understand how they change over time.

Can you give an example of a common related rates problem?

A common example is finding the rate at which the radius of a balloon is increasing when air is being pumped into it and the volume of the balloon is known.

What is the first step in solving a related rates worksheet problem?

The first step is to read the problem carefully to identify the quantities involved and the relationships between them.

Why do we use implicit differentiation in related rates?

Implicit differentiation is used because it allows us to differentiate equations that define relationships between variables without explicitly solving for one variable in terms of another.

How can units help in solving related rates problems?

Using units helps ensure that the rates of change are consistent and can prevent mistakes in calculations during the problem-solving process.

What are some common mistakes to avoid in related rates problems?

Common mistakes include forgetting to differentiate both sides of the equation, neglecting to convert units, or misinterpreting the relationships between quantities.

How do you find the rate of change of one variable with respect to another?

To find the rate of change of one variable with respect to another, differentiate the relationship equation with respect to time and then solve for the desired rate.

What resources can help with understanding related rates?

Resources such as calculus textbooks, online video tutorials, and practice worksheets specifically focused on related rates can be very helpful for understanding and mastering the topic.

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