Real Analysis Qualifying Exam Solutions

Real Analysis Qualifying Examination

Fall 2023

The five problems on this exam have equal weighting. To receive full credit give complete justification for all assertions by either citing known theorems or giving arguments from first principles.

1. Let $f_n(x) = \frac{nx^2}{n^3 + x^3}$.

   (a) Prove that f_n converge uniformly to 0 on $[0, M]$ for any $M > 0$, but does not converge uniformly to 0 on $[0, \infty)$.

   (b) Prove that the series $\sum_{n=1}^{\infty} f_n(x)$ defines a continuous function on $[0, \infty)$.

2. Let $(X, \mathcal{A})$ be a measurable space and μ is a non-negative set function on $\mathcal{A}$ that is finitely additive with $\mu(\emptyset) = 0$. Recall that such a set function is said to be *continuous from below* if

$$\mu\Big(\bigcup_j A_j\Big) = \lim_{j\to\infty} \mu(A_j) \quad \text{whenever } A_j \text{ is an increasing sequence of sets in } \mathcal{A}.$$

   Prove that

$$\mu \text{ is a measure} \iff \mu \text{ is continuous from below.}$$

3. Prove that

$$1 - \frac{x^2}{2} \le \cos x \le e^{-x^2/2}$$

   for all $|x| \le 1$ and conclude from this that

$$\lim_{n\to\infty} \sqrt{\frac{n}{2\pi}} \int_{|x|\le 1} (\cos x)^n \, dx = 1.$$

   Hint: You may use without proof that $\int_{-\infty}^{\infty} e^{-\pi x^2}\, dx = 1$.

4. Let $a, b > 0$. Prove that

$$\int_{[0,1]\times[0,1]} \frac{1}{x^a + y^b} \, dm_2(x, y) < \infty \iff \frac{1}{a} + \frac{1}{b} > 1$$

   where m_2 denotes Lebesgue measure on $\mathbb{R}^2$.

   Hint: One possible approach would be to consider separately the regions where $x^a \le y^b$ and $x^a > y^b$.

5. Let $f_k \to f$ a.e. on $\mathbb{R}$ with $\sup_k \|f_k\|_{L^2(\mathbb{R})} < \infty$. Prove that $f \in L^2(\mathbb{R})$ and that

$$\lim_{k\to\infty} \int_{\mathbb{R}} f_k g = \int_{\mathbb{R}} f g$$

   for all $g \in L^2(\mathbb{R})$.

   Hint: First consider functions g supported on sets of finite measure and use Egorov's Theorem.

Real analysis qualifying exam solutions are a crucial aspect of graduate-level mathematics education, particularly for students specializing in pure mathematics. These exams serve as a gatekeeping mechanism, assessing a student's understanding of foundational concepts in real analysis. This article delves into the structure and content of real analysis qualifying exams, common topics covered, strategies for preparation, and insights into effective solutions.

Understanding Real Analysis Qualifying Exams

Real analysis qualifying exams are typically designed to evaluate a student's grasp of essential topics such as sequences, series, continuity, differentiability, integration, and metric spaces. These exams often occur at the end of the first year of graduate school and are a prerequisite for advancing into more specialized areas of study.

Purpose of the Exam

The primary objectives of real analysis qualifying exams include:

- Assessing foundational knowledge in real analysis.
- Ensuring students have a solid understanding of proofs and theoretical concepts.
- Preparing students for more advanced coursework and research.

Structure of the Exam

Typically, real analysis qualifying exams consist of:

- A series of problems that test various topics in real analysis.
- A time limit that usually ranges from 3 to 6 hours.
- A mix of theoretical questions and practical problems requiring rigorous proofs.

Students are often required to demonstrate not only their ability to compute but also their capacity to articulate their reasoning and provide clear, logical arguments.

Common Topics Covered

The content of real analysis qualifying exams can vary by institution, but certain topics are universally recognized as essential. These include:

1. **Sequences and Series**
    - Convergence and divergence
    - Limit theorems
    - Power series
2. **Continuity**
    - Definitions and properties

- Uniform continuity
- Intermediate value theorem

3. **Differentiation**
   - Mean value theorem
   - Applications of derivatives
   - Functions of several variables

4. **Integration**
   - Riemann integrability
   - Fundamental theorem of calculus
   - Improper integrals

5. **Metric Spaces**
   - Definitions and examples
   - Open and closed sets
   - Compactness and connectedness

Strategies for Preparation

Preparing for a real analysis qualifying exam requires a structured approach and a deep engagement with the material. Here are some effective strategies:

Create a Study Schedule

A well-structured study schedule is vital. Allocate time for each topic based on its complexity and your familiarity with it. Consider the following steps:

- Divide your study time into blocks dedicated to specific topics.
- Review lecture notes and textbooks, focusing on definitions, theorems, and proofs.
- Practice problems regularly to reinforce the concepts.

Utilize Past Exam Papers

Working through past exam papers is one of the best ways to prepare. This practice allows you to:

- Familiarize yourself with the exam format.
- Identify recurring themes and types of problems.
- Gauge your time management skills under exam conditions.

Join Study Groups

Collaborating with peers can enhance your understanding and provide different perspectives on challenging concepts. Benefits of study groups include:

- Discussion of difficult topics and clarification of doubts.
- Sharing solutions and strategies for approaching problems.
- Motivation and support from fellow students.

Seek Help from Professors and TAs

Don’t hesitate to reach out to your professors or teaching assistants for guidance. They can provide valuable insights on:

- Key topics to focus on for the exam.
- Clarifications on complex concepts.
- Recommended resources for further studying.

Effective Solutions to Exam Problems

When tackling problems on real analysis qualifying exams, the quality of your solutions matters as much as your understanding of the material. Here are some tips for crafting effective solutions:

Read the Questions Carefully

Taking the time to understand exactly what is being asked is crucial. Misinterpretations can lead to incorrect solutions. Here’s how to ensure clarity:

- Identify keywords that indicate the type of proof or computation required (e.g., "prove," "show," "find").
- Break down complex problems into manageable parts.

Write Clear and Concise Proofs

A well-structured proof is essential in real analysis. Follow these guidelines:

- Start with definitions relevant to your proof.
- Clearly state your assumptions and what you aim to prove.
- Use logical progression: Each step should naturally lead to the next.
- Conclude with a summary statement that ties back to the original question.

Include Justifications for Each Step

In real analysis, providing justifications for each step in your solution is vital. Use:

- Theorems and lemmas to justify your arguments.
- Examples to illustrate concepts when appropriate.
- Counterexamples to clarify why certain statements are false.

Practice Time Management

During the exam, allocate your time wisely:

- Spend no more than 25% of your time on the first few questions, ensuring you move on to others.
- Return to challenging problems after addressing those you can solve quickly.

Conclusion

In summary, real analysis qualifying exam solutions represent not just the answers to problems but the culmination of a student’s understanding of complex mathematical principles. By familiarizing themselves with common topics, applying effective study strategies, and practicing clear and logical problem-solving techniques, students can significantly enhance their chances of success. The journey through real analysis is challenging but ultimately rewarding, paving the way for advanced studies and a deeper appreciation of the mathematical sciences.

Frequently Asked Questions

What is a typical format of real analysis qualifying exam questions?

Real analysis qualifying exam questions often include proofs of theorems, counterexamples, and problems related to sequences, series, continuity, differentiation, and integration of functions.

How should I prepare for real analysis qualifying exams?

To prepare, review key concepts in real analysis, solve previous exam papers, understand proofs in detail, and practice writing clear and rigorous arguments.

What topics are commonly covered in real analysis qualifying exams?

Common topics include limits, continuity, differentiability, Riemann integration, metric spaces, and convergence of sequences and series.

Are solutions to past real analysis qualifying exams available online?

Yes, many universities provide solutions to past qualifying exams on their websites or through departmental resources, which can be invaluable for study.

What are some common mistakes to avoid in real analysis exams?

Common mistakes include misunderstanding definitions, making logical leaps in proofs, and neglecting to justify every step in arguments.

How important is it to understand the proofs of theorems in real analysis?

Understanding the proofs is crucial as they form the foundation of the subject and are often a focus in qualifying exams.

What resources are recommended for studying real analysis?

Recommended resources include textbooks such as 'Principles of Mathematical Analysis' by Rudin, lecture notes from university courses, and online platforms like Coursera and MIT OpenCourseWare.

Can group study help in preparing for real analysis qualifying exams?

Yes, group study can be beneficial as discussing concepts and solving problems collaboratively can enhance understanding and retention of material.

What is the best strategy for managing time during a real analysis exam?

The best strategy includes skimming through all questions first, allocating time based on the difficulty of each question, and ensuring to leave time for review and checking answers.

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