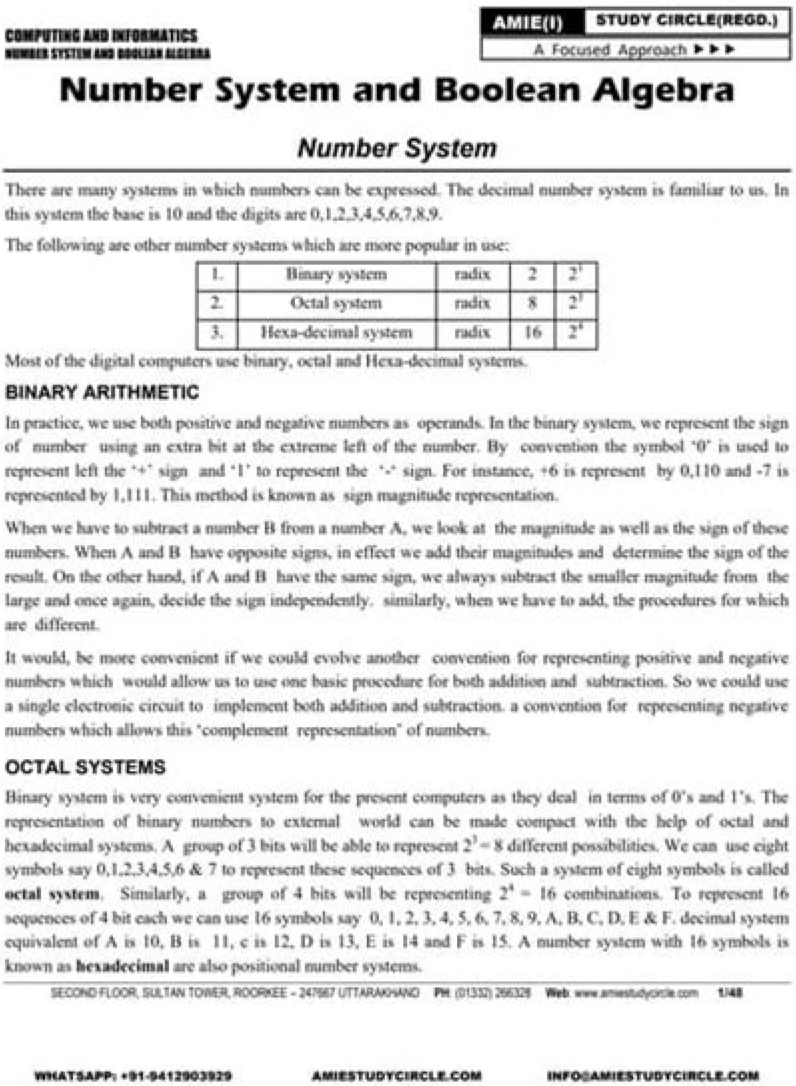
Number System And Boolean Algebra



COMPUTING AND INFORMATICS
NUMBER SYSTEM AND BOOLEAN ALGEBRA

AMIE(I) STUDY CIRCLE(REGD.)
A Focused Approach ►►►

Number System and Boolean Algebra

Number System

There are many systems in which numbers can be expressed. The decimal number system is familiar to us. In this system the base is 10 and the digits are 0,1,2,3,4,5,6,7,8,9.

The following are other number systems which are more popular in use:

1.	Binary system	radix	2	2^1
2.	Octal system	radix	8	2^3
3.	Hexa-decimal system	radix	16	2^4

Most of the digital computers use binary, octal and Hexa-decimal systems.

BINARY ARITHMETIC

In practice, we use both positive and negative numbers as operands. In the binary system, we represent the sign of number using an extra bit at the extreme left of the number. By convention the symbol '0' is used to represent left the '+' sign and '1' to represent the '-' sign. For instance, +6 is represent by 0,110 and -7 is represented by 1,111. This method is known as sign magnitude representation.

When we have to subtract a number B from a number A, we look at the magnitude as well as the sign of these numbers. When A and B have opposite signs, in effect we add their magnitudes and determine the sign of the result. On the other hand, if A and B have the same sign, we always subtract the smaller magnitude from the large and once again, decide the sign independently. similarly, when we have to add, the procedures for which are different.

It would, be more convenient if we could evolve another convention for representing positive and negative numbers which would allow us to use one basic procedure for both addition and subtraction. So we could use a single electronic circuit to implement both addition and subtraction. a convention for representing negative numbers which allows this 'complement representation' of numbers.

OCTAL SYSTEMS

Binary system is very convenient system for the present computers as they deal in terms of 0's and 1's. The representation of binary numbers to external world can be made compact with the help of octal and hexadecimal systems. A group of 3 bits will be able to represent $2^3 = 8$ different possibilities. We can use eight symbols say 0,1,2,3,4,5,6 & 7 to represent these sequences of 3 bits. Such a system of eight symbols is called **octal system**. Similarly, a group of 4 bits will be representing $2^4 = 16$ combinations. To represent 16 sequences of 4 bit each we can use 16 symbols say 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E & F. decimal system equivalent of A is 10, B is 11, c is 12, D is 13, E is 14 and F is 15. A number system with 16 symbols is known as **hexadecimal** are also positional number systems.

SECOND FLOOR, SULTAN TOWER, ROORKEE – 247667 UTTARAKHAND PH: (01332) 266328 Web: www.amiestudycircle.com 1/48

WHATSAPP: +91-9412903929 AMIESTUDYCIRCLE.COM INFO@AMIESTUDYCIRCLE.COM

Number system and Boolean algebra are foundational concepts in mathematics and computer science, playing a critical role in how we represent and manipulate data. The number system is a structured way of expressing quantities, while Boolean algebra provides a framework for reasoning about logical propositions. Together, they form the bedrock for numerous applications in digital electronics, programming, and algorithm design. Understanding these concepts is essential for anyone looking to delve into computing and data analysis.

Understanding the Number System

The number system is a way of expressing and categorizing numbers based on certain properties. It includes various types of numbers, each serving different purposes in mathematics and its applications. The most commonly used number systems are:

1. Natural Numbers

Natural numbers are the simplest form of numbers used for counting and ordering. They start from 1 and go on indefinitely (1, 2, 3, ...).

- Properties:
- They are positive integers.
- They do not include zero or negative numbers.
- They can be used in basic arithmetic operations like addition and multiplication.

2. Whole Numbers

Whole numbers include all natural numbers and the number zero.

- Properties:
- They are non-negative integers (0, 1, 2, 3, ...).
- They are used in scenarios where zero is a meaningful quantity, such as representing the absence of objects.

3. Integers

Integers expand the number system to include negative numbers, forming a complete set that includes positive numbers, negative numbers, and zero.

- Properties:
- They include {..., -3, -2, -1, 0, 1, 2, 3, ...}.
- They are useful in situations requiring a measure of debt or loss alongside gain.

4. Rational Numbers

Rational numbers are numbers that can be expressed as a fraction of two integers, where the denominator is not zero.

- Examples:
- 1/2, -3/4, 5 (which can be expressed as 5/1).
- Properties:
- They can be finite or recurring decimals.
- They are useful in calculations involving parts of whole quantities.

5. Irrational Numbers

Irrational numbers cannot be expressed as a simple fraction. They have decimal expansions that are non-repeating and non-terminating.

- Examples:
- $\sqrt{2}$, π, e.
- Properties:
- They fill the gaps between rational numbers on the number line.
- They are important in advanced mathematics, particularly in geometry and calculus.

6. Real Numbers

Real numbers encompass all rational and irrational numbers, representing all points on the number line.

- Properties:
- They include both finite and infinite sets of numbers.
- They are crucial in various mathematical analyses, especially in calculus.

7. Complex Numbers

Complex numbers include a real part and an imaginary part, often expressed in the form a + bi, where "i" is the imaginary unit.

- Examples:
- 3 + 4i, where 3 is the real part, and 4i is the imaginary part.
- Properties:
- They are used in advanced mathematics and engineering, particularly in fields like electrical engineering and quantum physics.

Boolean Algebra Basics

Boolean algebra, named after mathematician George Boole, is a branch of algebra that deals with variables that have two possible values: true and false (or 1 and 0). It is fundamental in computer science, particularly in logic design, computer programming, and set theory.

1. Basic Operations

Boolean algebra operates on binary values using a set of basic logical operations:

- AND (Conjunction): The result is true only if both operands are true.
- Symbol: A · B
- Truth Table:
- A = 0, B = 0 → Result = 0
- A = 0, B = 1 → Result = 0
- A = 1, B = 0 → Result = 0
- A = 1, B = 1 → Result = 1

- OR (Disjunction): The result is true if at least one operand is true.
- Symbol: A + B
- Truth Table:
- A = 0, B = 0 → Result = 0
- A = 0, B = 1 → Result = 1
- A = 1, B = 0 → Result = 1
- A = 1, B = 1 → Result = 1

- NOT (Negation): The result is the opposite of the operand.
- Symbol: ¬A or A'
- Truth Table:
- A = 0 → Result = 1
- A = 1 → Result = 0

2. Laws of Boolean Algebra

Boolean algebra is governed by several fundamental laws that simplify logical expressions:

- Identity Law:
- A + 0 = A
- A · 1 = A

- Null Law:
- A + 1 = 1
- A · 0 = 0

- Idempotent Law:
- A + A = A
- A · A = A

- Complement Law:
- A + ¬A = 1
- A · ¬A = 0

- Distributive Law:
- A · (B + C) = (A · B) + (A · C)
- A + (B · C) = (A + B) · (A + C)

3. Applications of Boolean Algebra

Boolean algebra finds extensive applications across various fields, especially in computer science:

- Digital Circuit Design: Boolean expressions are used to design and simplify digital circuits, enabling efficient electronic systems.
- Search Algorithms: In databases and search engines, Boolean logic is applied for query processing.
- Programming: Conditional statements in programming languages utilize Boolean logic to determine the flow of control.

Interplay between Number Systems and Boolean Algebra

The interaction between number systems and Boolean algebra is crucial in various computing tasks. Digital computers operate using binary numbers, a form of the number system that utilizes only two digits: 0 and 1. This binary representation aligns seamlessly with Boolean algebra, where logical operations can be directly mapped to binary operations.

1. Binary Number System

The binary number system is a base-2 numeral system that uses only two digits, 0 and 1. It is the foundation of modern computing and digital electronics.

- Example: The decimal number 5 is represented in binary as 101.

2. Logic Gates and Circuits

Logic gates, which implement Boolean functions, are the building blocks of digital circuits. Each gate corresponds to a specific Boolean operation.

- Types of Logic Gates:
- AND Gate
- OR Gate
- NOT Gate
- NAND Gate
- NOR Gate
- XOR Gate

These gates process binary numbers and perform arithmetic operations, fulfilling the requirements of complex computations.

3. Converting Between Number Systems

Understanding how to convert between different number systems is essential, particularly when dealing with digital data.

- Decimal to Binary: A method involves dividing the decimal number by 2 and recording remainders.
- Binary to Decimal: Each bit in a binary number represents a power of 2, and the sum of these powers gives the decimal equivalent.

Conclusion

Mastering the number system and Boolean algebra is vital for anyone interested in mathematics, computer science, or engineering. These concepts provide essential tools for representing and manipulating data, designing algorithms, and creating efficient digital systems. As technology continues to evolve, the relevance of these mathematical foundations will only grow, paving the way for advancements in artificial intelligence, data science, and beyond. Understanding these principles not only enhances one's mathematical proficiency but also opens doors to innovative problem-solving techniques in various fields.

Frequently Asked Questions

What are the main types of number systems used in computing?

The main types of number systems used in computing are binary (base 2), decimal (base 10), octal (base 8), and hexadecimal (base 16).

How is binary arithmetic different from decimal arithmetic?

Binary arithmetic uses only two digits, 0 and 1, while decimal arithmetic uses ten digits, 0-9. This results in different rules for carrying over and borrowing during addition and subtraction.

What is Boolean algebra and why is it important in computer science?

Boolean algebra is a branch of algebra that deals with true/false values and is fundamental in computer science for designing circuits, algorithms, and performing logical operations.

Can you explain the difference between 'AND', 'OR', and 'NOT' operations in Boolean algebra?

'AND' returns true only if both operands are true, 'OR' returns true if at least one operand is true, and 'NOT' inverts the truth value of its operand.

What is a truth table and how is it used in Boolean algebra?

A truth table is a mathematical table used to determine the output of a Boolean expression for all possible input values, helping to visualize and analyze logical operations.

How can you convert a decimal number to binary?

To convert a decimal number to binary, repeatedly divide the number by 2 and record the remainders. The binary representation is formed by reading the remainders in reverse order.

What is the significance of the binary number system in digital electronics?

The binary number system is significant in digital electronics because it aligns with the two states of electronic components (on/off, high/low), making it ideal for representing data in computers.

What are Karnaugh maps and how do they relate to Boolean algebra?

Karnaugh maps are graphical tools used to simplify Boolean expressions and minimize logic circuits by visually grouping together terms that produce the same output.

Find other PDF article:
https://soc.up.edu.ph/54-tone/files?ID=jWw30-5334&title=smoldering-ashes-diablo-4-guide.pdf

Number System And Boolean Algebra

odd number□even number□□□□□ - □□□□
odd number□even number□□□□□□□□□□□□□odd number□even number□□□□□□odd number□□□□noun□odd number□□□□□□□□□□□□□□□□□2□□□□□

□□□VAT number_□□□□
vat number□□□□Value Added Tax□□vat□□□□□□□□□□□□□vat number□□□□□□□□□□□□ □□ ...

part number□□□□_□□□□
Jun 4, 2024 · part number□□□□part number□□□□□□“□□□□”□ □□□□□□□□□□□□part number□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

numerals□number□□□_□□□□
□□□numeral□□□number□□□□□□□□□□□□□□□□□ Numeral□□□□represent□□□□□□□□□□□□□□□□□writing system□□□□□□□ …

numbers of □ *a number of*□□□ - □□□□
Sep 18, 2006 ·)a number of□□“□□□□□□”□number□□□large□□□of□□□□□□□□□□□large numbers of, □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

□□□□□□□serial number? - □□□□
□□□□□□□serial number?□□□□□□□□□□□□□□□□□SN□□□Serial Number□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

□□□ *number one* □□□□ *NO.1*□*NO.* □□□□□□□□□ *…*
Aug 2, 2016 · Number Ten announced tonight that the election will be on April 6. □□□□□□□□□□□□□□4□6□□□□ 3 any number of things□□□□□ □□□ The Cook's Shop …

Account number□IBAN□□□□□? - □□
May 14, 2021 · □□□□□□IBAN□□□□□□□□□□□□□□□□□□account number □□□□□?

amount□number □ quantity □□□□□□□ - □□□□
May 24, 2017 · amount□number□quantity□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ □□□□□□□□□ 1.amount□□□□□;□□;□□ 2.number□□□□□;□;□□; …

VAT□□□□□□□*Tax number*□□□□□□_□□□□
Nov 8, 2024 · VAT (Value Added Tax)□Tax Number□□□□□□□□□□□□□□□□□□□□□□□□□VAT□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

odd number□even number□□□□□ - □□□□
odd number□even number□□□□□□□□□□□□odd number□even number□□□□□odd number□□□□noun□odd number□□□□□□□□□□□□□□2□□□□□

□□□VAT number_□□□□
vat number□□□□Value Added Tax□□vat□□□□□□□□□□□vat number□□□□□□□□□□□ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

part number□□□□_□□□□
Jun 4, 2024 · part number□□□□part number□□□□□□“□□□□”□ □□□□□□□□□□□□part number□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

numerals□number□□□_□□□□
□□□numeral□□□number□□□□□□□□□□□□□□□□□ Numeral□□□□represent□□□□□□□□□□□□□□□□□writing system□□□□□□□ …

numbers of □ a number of□□□ - □□□□
Sep 18, 2006 ·)a number of□□“□□□□□□”□number□□□large□□□of□□□□□□□□□□□large numbers of, □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ …

[illegible]serial number? - [illegible]
[illegible]serial number?[illegible]SN[illegible]Serial Number[illegible] …

[illegible] number one [illegible] NO.1[illegible]NO. [illegible] …
Aug 2, 2016 · Number Ten announced tonight that the election will be on April 6. [illegible]4[illegible]6[illegible] 3 any number of things[illegible] [illegible] The Cook's Shop …

Account number[illegible]IBAN[illegible]? - [illegible]
May 14, 2021 · [illegible]IBAN[illegible]account number [illegible]?

amount[illegible]number [illegible] quantity [illegible] - [illegible]
May 24, 2017 · amount[illegible]number[illegible]quantity[illegible] [illegible] 1.amount[illegible];[illegible];[illegible] 2.number[illegible];[illegible];[illegible]; …

VAT[illegible]Tax number[illegible]_[illegible]
Nov 8, 2024 · VAT (Value Added Tax)[illegible]Tax Number[illegible]VAT[illegible] …

Explore the fundamentals of the number system and Boolean algebra. Understand their significance in computing and logic. Discover how they shape technology today!

[Back to Home]