

Maths Olympiad Problems And Solutions

Solutions

Twentieth International Olympiad, 1978

1978/1. Since 1978^n and 1978^m agree in their last three digits, the difference

$$1978^n - 1978^m = 1978^m(1978^{n-m} - 1)$$

is divisible by $10^3 = 2^3 \cdot 5^3$; and since the second factor above is odd, 2^3 divides the first. Also

$$1978^m = 2^m \cdot 989^m,$$

so $m \geq 3$.

We can write $m + n = (n - m) + 2m$; to minimize this sum, we take $m = 3$ and seek the smallest value of $d = n - m$, such that $1978^d - 1$ is divisible by $5^3 = 125$, i.e.

$$1978^d \equiv 1 \pmod{125}.$$

We shall twice make use of the following

LEMMA. Let d be the smallest exponent such that $a^d \equiv 1 \pmod{N}$. Then any other exponent g for which $a^g \equiv 1 \pmod{N}$ is a multiple of d .

PROOF: If d does not divide g , then $g = qd + r$ with $0 < r < d$, and $a^g = a^{qd}a^r \equiv 1 \pmod{N}$ implies $a^r \equiv 1 \pmod{N}$ with $0 < r < d$, contradicting the minimality of d . So $d|g$.

Fermat's theorem states:† For any prime p and any integer a not divisible by p ,

$$(1) \quad a^{p-1} \equiv 1 \pmod{p}.$$

For example,

$$1978^4 \equiv 1 \pmod{5}.$$

†For a proof, see e.g. p. 126 of S. L. Greitzer, *The International Mathematical Olympiads*, vol. 27 in this NML series.

Maths Olympiad problems and solutions are a central focus for many students who aspire to excel in mathematics competitions. These problems not only challenge the intellect but also foster critical thinking, creativity, and problem-solving skills among young mathematicians. In this article, we will delve into the nature of Maths Olympiad problems, explore various strategies for solving them, and provide a selection of problems along with their detailed solutions.

Understanding Maths Olympiad Problems

Maths Olympiad problems are designed to test a wide range of mathematical concepts, including but not limited to:

- Algebra: Solving equations, inequalities, and working with polynomials.
- Geometry: Understanding shapes, theorems, and properties of space.
- Number Theory: Investigating integers, divisibility, and prime numbers.
- Combinatorics: Counting methods, permutations, and combinations.
- Logic: Deductive reasoning and problem-solving strategies.

These problems often require a deep understanding of the underlying principles and the ability to apply them in novel ways.

The Format of Maths Olympiad Problems

Typically, Maths Olympiad problems are presented in various formats, including:

1. Multiple Choice Questions: Where participants select the correct answer from given options.
2. Open-Ended Questions: Where the participants must derive a solution and provide a detailed explanation.
3. Proof-Based Questions: These require participants to prove a mathematical statement or theorem.

The difficulty of these problems varies widely, with some designed for junior participants and others aimed at advanced competitors.

Techniques for Solving Maths Olympiad Problems

To tackle Maths Olympiad problems effectively, students can employ several strategies:

1. Master the Fundamentals

A strong grasp of basic mathematical concepts is essential. Students should ensure they understand:

- Fundamental theorems in algebra and geometry.
- Basic properties of numbers in number theory.
- Essential counting principles in combinatorics.

2. Practice Regularly

Regular practice with past Olympiad problems can significantly enhance problem-solving skills. Students should:

- Solve a variety of problems from previous Olympiads.
- Engage with online platforms or local math clubs for additional resources.

3. Explore Different Approaches

Many problems can be solved in multiple ways. Students should:

- Experiment with various techniques, such as drawing diagrams for geometry problems or using algebraic manipulations.
- Consider both direct and indirect methods to reach a solution.

4. Time Management

During competitions, time is of the essence. Students should:

- Practice solving problems within a set time limit.
- Learn to identify which problems to tackle first based on their strengths and weaknesses.

5. Collaborate and Discuss

Working with peers can provide new perspectives. Students should:

- Join study groups or forums to discuss challenging problems.
- Share ideas and solutions to foster a collaborative learning environment.

Sample Maths Olympiad Problems and Solutions

Here, we present a selection of Maths Olympiad problems along with their comprehensive solutions to illustrate the techniques discussed.

Problem 1: The Geometry Challenge

Problem: In triangle ABC, the lengths of sides AB and AC are 7 cm and 10 cm, respectively. If the angle A measures 60 degrees, find the length of side BC.

Solution:

To solve this, we can use the Law of Cosines, which states that:

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

Here, $a = 7$, $b = 10$, and $C = 60^\circ$.

1. Substitute the values into the formula:

$$BC^2 = 7^2 + 10^2 - 2 \cdot 7 \cdot 10 \cdot \cos(60^\circ)$$

2. Simplify:

$$BC^2 = 49 + 100 - 140 \cdot \frac{1}{2}$$

$$BC^2 = 149 - 70 = 79$$

$$BC = \sqrt{79} \approx 8.89 \text{ cm}$$

Thus, the length of side BC is approximately 8.89 cm.

Problem 2: The Number Theory Enigma

Problem: Find the smallest positive integer (n) such that $(n \equiv 2 \pmod{3})$ and $(n \equiv 3 \pmod{4})$.

Solution:

To solve this system of congruences, we can use the method of successive substitutions.

1. Start with the first congruence $(n \equiv 2 \pmod{3})$:

This means $(n = 3k + 2)$ for some integer (k) .

2. Substitute into the second congruence $(n \equiv 3 \pmod{4})$:

$$3k + 2 \equiv 3 \pmod{4}$$

Simplifying gives:

$$3k \equiv 1 \pmod{4}$$

3. The multiplicative inverse of 3 modulo 4 is 3, since $(3 \cdot 3 \equiv 1 \pmod{4})$. Thus, we can multiply both sides of the equation by 3:

$$k \equiv 3 \pmod{4}$$

\]

Therefore, $k = 4m + 3$ for some integer m .

4. Substitute back to find n :

\[

$$n = 3(4m + 3) + 2 = 12m + 9 + 2 = 12m + 11$$

\]

5. The smallest positive integer occurs when $m = 0$:

\[

$$n = 11$$

\]

Thus, the smallest positive integer n that satisfies both congruences is $n = 11$.

Problem 3: The Combinatorial Conundrum

Problem: How many ways can 4 different books be arranged on a shelf?

Solution:

To determine the number of ways to arrange 4 different books, we can use the formula for permutations, which is given by $n!$ (n factorial).

1. For 4 books, the number of arrangements is:

\[

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

\]

Thus, there are 24 different ways to arrange 4 different books on a shelf.

Conclusion

Maths Olympiad problems and solutions provide an exciting and challenging avenue for students to engage with mathematics. Through understanding the types of problems, employing effective strategies, and practicing regularly, students can develop their mathematical abilities and excel in competitions. The problems presented in this article are just a small sample of the rich variety of challenges that await aspiring mathematicians. With dedication and perseverance, students can enjoy the journey of discovery that mathematics offers.

Frequently Asked Questions

What are the common types of problems found in Maths Olympiads?

Common types of problems include number theory, combinatorics, geometry, and algebra. Each category often requires creative problem-solving skills and a deep understanding of mathematical concepts.

How can students prepare for Maths Olympiad competitions?

Students can prepare by practicing past Olympiad problems, studying advanced topics beyond the school curriculum, joining math clubs, and participating in mock competitions to enhance their problem-solving skills.

What role do logic and reasoning play in solving Maths Olympiad problems?

Logic and reasoning are crucial in Maths Olympiad problems as they often require the ability to deduce relationships, formulate strategies, and apply mathematical theories to arrive at solutions.

Are there any recommended resources for finding Maths Olympiad problems?

Yes, recommended resources include books like 'The Art and Craft of Problem Solving' by Paul Zeitz, online platforms like AoPS (Art of Problem Solving), and past papers from national and international Olympiads.

What strategies can be used to approach difficult Maths Olympiad problems?

Strategies include breaking down the problem into smaller parts, looking for patterns, working backward from the solution, and considering alternative methods of solving the problem.

How important is time management during a Maths Olympiad?

Time management is extremely important during a Maths Olympiad as many participants struggle to complete all problems within the time limit. Practicing under timed conditions can help improve speed and efficiency.

What is the significance of collaboration in preparing for Maths Olympiads?

Collaboration is significant as it allows students to share different problem-solving approaches, learn from one another, and tackle challenging problems together, enhancing their understanding and skills.

Can technology assist in preparing for Maths Olympiads?

Yes, technology can assist through online courses, interactive problem-solving platforms, and apps that provide practice problems and solutions, making learning more engaging and accessible.

How to deal with frustration when stuck on a Maths Olympiad problem?

When stuck on a problem, it's helpful to take a break, revisit the problem later with a fresh perspective, discuss it with peers, or tackle simpler related problems to build confidence and insight.

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