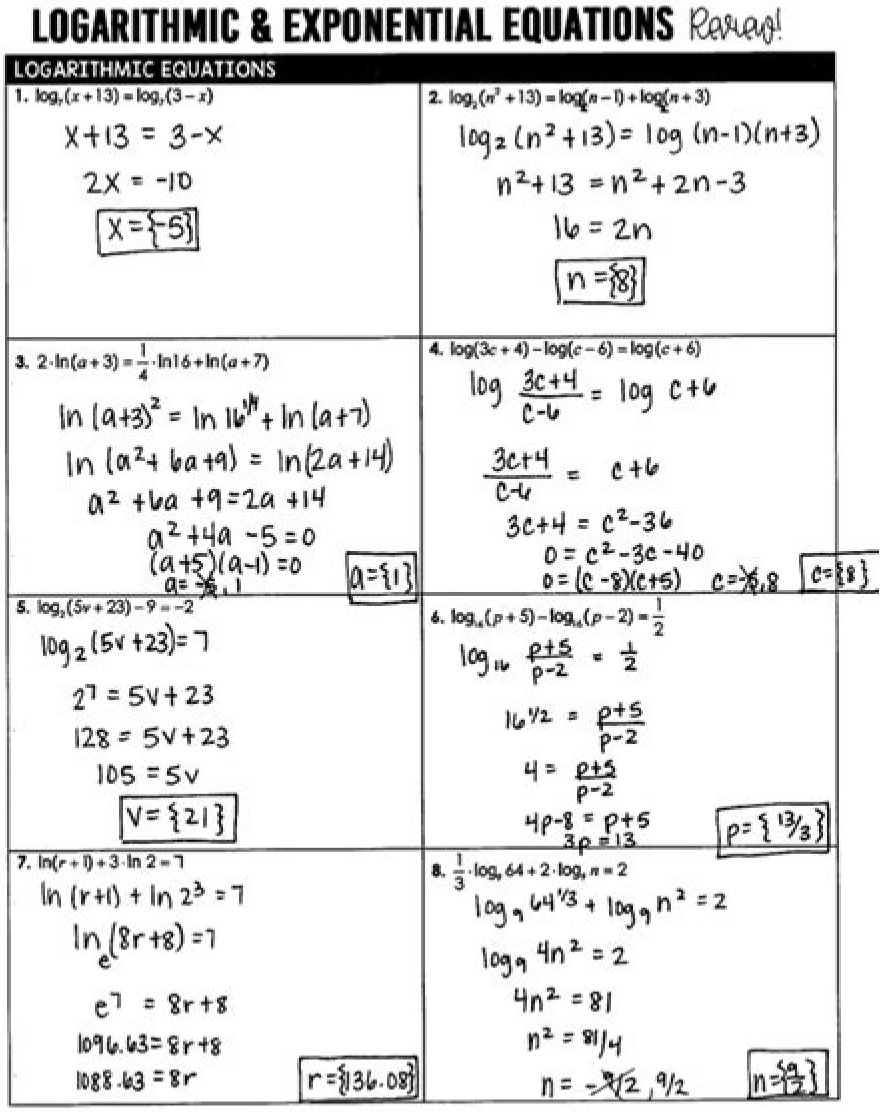
Math 120 Review Sheet Exponential And Logarithmic Functions


LOGARITHMIC & EXPONENTIAL EQUATIONS

LOGARITHMIC EQUATIONS

1. $\log_7(x+13) = \log_7(3-x)$

$x + 13 = 3 - x$

$2x = -10$

$x = \{-5\}$

2. $\log_2(n^2+13) = \log_2(n-1) + \log_2(n+3)$

$\log_2(n^2+13) = \log(n-1)(n+3)$

$n^2 + 13 = n^2 + 2n - 3$

$16 = 2n$

$n = \{8\}$

3. $2 \cdot \ln(a+3) = \frac{1}{4} \cdot \ln 16 + \ln(a+7)$

$\ln(a+3)^2 = \ln 16^{1/4} + \ln(a+7)$

$\ln(a^2+6a+9) = \ln(2a+14)$

$a^2 + 6a + 9 = 2a + 14$

$a^2 + 4a - 5 = 0$

$(a+5)(a-1) = 0$

$a = -5, 1$

$a = \{1\}$

4. $\log(3c+4) - \log(c-6) = \log(c+6)$

$\log \frac{3c+4}{c-6} = \log c+6$

$\frac{3c+4}{c-6} = c+6$

$3c + 4 = c^2 - 36$

$0 = c^2 - 3c - 40$

$0 = (c-8)(c+5)$ $c = -5, 8$

$c = \{8\}$

5. $\log_2(5v+23) - 9 = -2$

$\log_2(5v+23) = 7$

$2^7 = 5v + 23$

$128 = 5v + 23$

$105 = 5v$

$v = \{21\}$

6. $\log_{16}(p+5) - \log_{16}(p-2) = \frac{1}{2}$

$\log_{16} \frac{p+5}{p-2} = \frac{1}{2}$

$16^{1/2} = \frac{p+5}{p-2}$

$4 = \frac{p+5}{p-2}$

$4p - 8 = p + 5$

$3p = 13$

$p = \{13/3\}$

7. $\ln(r+1) + 3 \cdot \ln 2 = 7$

$\ln(r+1) + \ln 2^3 = 7$

$\ln(8r+8) = 7$

$e^7 = 8r + 8$

$1096.63 = 8r + 8$

$1088.63 = 8r$

$r = \{136.08\}$

8. $\frac{1}{3} \cdot \log_9 64 + 2 \cdot \log_9 n = 2$

$\log_9 64^{1/3} + \log_9 n^2 = 2$

$\log_9 4n^2 = 2$

$4n^2 = 81$

$n^2 = 81/4$

$n = -9/2, 9/2$

$n = \{9/2\}$

Math 120 Review Sheet: Exponential and Logarithmic Functions

Mathematics is a subject that builds upon itself, and understanding the foundations of exponential and logarithmic functions is vital for success in higher-level math courses. In Math 120, students delve into these functions, exploring their properties, applications, and relationships. This review sheet aims to provide a comprehensive overview of exponential and logarithmic functions, covering the key concepts and formulas that students need to know.

Understanding Exponential Functions

Exponential functions are mathematical expressions in which a constant base is raised to a variable exponent. The general form of an exponential function is:

$$ f(x) = a \cdot b^x $$

Where:
- $f(x)$ is the value of the function,
- a is a constant (the initial value or y-intercept),
- b is the base (a positive real number),
- x is the exponent.

Key Properties of Exponential Functions

1. Base Greater Than One: If $b > 1$, the function is increasing. This means as x increases, $f(x)$ also increases.
2. Base Between Zero and One: If $0 < b < 1$, the function is decreasing. As x increases, $f(x)$ decreases.
3. Intercepts: The y-intercept occurs at $(0, a)$. Exponential functions always pass through this point.
4. Horizontal Asymptote: The horizontal asymptote of exponential functions is $y = 0$ as x approaches negative infinity.
5. Domain and Range:
- Domain: All real numbers $(-\infty, +\infty)$
- Range: $(0, +\infty)$ if $a > 0$

Applications of Exponential Functions

Exponential functions model a variety of real-world phenomena, including:
- Population growth
- Radioactive decay
- Compound interest
- Natural phenomena like carbon dating

The formula for compound interest is particularly important:

$$ A = P(1 + \frac{r}{n})^{nt} $$

Where:
- A is the amount of money accumulated after n years, including interest.
- P is the principal amount (the initial sum of money).
- r is the annual interest rate (decimal).
- n is the number of times that interest is compounded per year.
- t is the number of years the money is invested or borrowed.

Understanding Logarithmic Functions

Logarithmic functions are the inverses of exponential functions. The general form of a logarithmic function is:

$$g(x) = \log_b(x)$$

Where:
- $g(x)$ is the logarithm of x to the base b,
- b is the base (a positive real number, $b \neq 1$),
- x is the argument of the logarithm (a positive real number).

Key Properties of Logarithmic Functions

1. Inverse Relationship: If $f(x) = b^x$, then $g(x) = \log_b(x)$. This means $f(g(x)) = x$ and $g(f(x)) = x$.
2. Domain and Range:
- Domain: $(0, +\infty)$
- Range: All real numbers $(-\infty, +\infty)$
3. Intercept: The x-intercept occurs at $(1, 0)$ since $\log_b(1) = 0$.
4. Vertical Asymptote: The vertical asymptote is $x = 0$ as x approaches zero from the right.
5. Growth: Logarithmic functions grow slowly compared to exponential functions.

Applications of Logarithmic Functions

Logarithmic functions are used in various applications including:
- Measuring pH in chemistry,
- The Richter scale for earthquake magnitude,
- Decibels in acoustics,
- Financial modeling for calculating time to reach a certain investment value.

Change of Base Formula

When working with logarithms, you may need to change the base of a logarithm. The change of base formula is given by:

$$\log_b(a) = \frac{\log_k(a)}{\log_k(b)}$$

Where k is any positive number (commonly 10 or e for natural logarithms). This formula allows you to compute logarithms with different bases using a calculator.

Key Logarithmic and Exponential Identities

Understanding certain identities can greatly simplify calculations involving exponential and logarithmic functions. Here are some important identities:

Exponential Identities

1. $b^{\log_b(x)} = x$
2. $\log_b(b^x) = x$
3. $b^{x+y} = b^x \cdot b^y$
4. $b^{x-y} = \frac{b^x}{b^y}$

Logarithmic Identities

1. $\log_b(xy) = \log_b(x) + \log_b(y)$
2. $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
3. $\log_b(x^r) = r \cdot \log_b(x)$

Solving Exponential and Logarithmic Equations

Solving equations involving exponential and logarithmic functions is a critical skill in Math 120. Here are methods to approach these types of equations:

Solving Exponential Equations

1. Isolate the exponential expression: Start by getting the exponential term alone on one side.
2. Take the logarithm of both sides: Use natural logarithm $\ln$ or common logarithm $\log$.
3. Apply logarithmic identities: Utilize properties of logarithms to simplify.
4. Solve for the variable: Isolate the variable to find the solution.

Example:
To solve $3^x = 81$:
- Rewrite 81 as 3^4.
- Thus, $3^x = 3^4$ implies $x = 4$.

Solving Logarithmic Equations

1. Isolate the logarithmic expression: Get the logarithm alone on one side of the equation.
2. Exponentiate both sides: Use the base of the logarithm to eliminate the logarithm.
3. Solve for the variable: Isolate the variable to determine the solution.

Example:
To solve $\log_2(x) = 3$:
- Exponentiate: $x = 2^3 = 8$.

Graphing Exponential and Logarithmic Functions

Graphing these functions helps visualize their behavior.

Graphing Exponential Functions

- The graph of $f(x) = b^x$ has a horizontal asymptote at $y = 0$.
- As x increases, $f(x)$ increases rapidly if $b > 1$ or decreases if $0 < b < 1$.
- Example: For $f(x) = 2^x$, the graph passes through $(0, 1)$ and increases steeply.

Graphing Logarithmic Functions

- The graph of $g(x) = \log_b(x)$ has a vertical asymptote at $x = 0$.
- The function increases slowly and passes through $(1, 0)$.
- Example: For $g(x) = \log_2(x)$, the graph rises gradually and never touches the y-axis.

Conclusion

Mastering exponential and logarithmic functions is essential for success in Math 120 and beyond. Understanding their definitions, properties, applications, and solving techniques provides a solid foundation for tackling more complex mathematical concepts. This review sheet serves as a guide to reinforce these critical concepts, enabling students to approach their studies with confidence. Remember to practice solving equations, graphing functions, and applying the properties discussed to solidify your understanding and prepare for exams.

Frequently Asked Questions

What is the basic definition of an exponential function?

An exponential function is a mathematical function of the form $f(x) = a b^x$, where a is a constant, b is the base (a positive real number not equal to 1), and x is the exponent.

How do you convert an exponential equation to a logarithmic form?

To convert an exponential equation of the form $b^y = x$ to logarithmic form, you rewrite it as $\log_b(x)$

= y, where b is the base.

What is the natural logarithm and how is it denoted?

The natural logarithm is the logarithm to the base e, where e is approximately 2.718. It is denoted as ln(x).

What is the relationship between exponential and logarithmic functions?

Exponential and logarithmic functions are inverse functions. This means that if y = b^x, then x = log_b(y), and vice versa.

How do you solve an exponential equation like 3^(2x) = 81?

To solve 3^(2x) = 81, you can rewrite 81 as 3^4. Therefore, 2x = 4, which gives x = 2.

What properties of logarithms can be used to simplify log expressions?

The properties of logarithms include the product rule (log_b(xy) = log_b(x) + log_b(y)), the quotient rule (log_b(x/y) = log_b(x) - log_b(y)), and the power rule (log_b(x^k) = k log_b(x)).

What is the domain and range of the exponential function f(x) = 2^x?

The domain of f(x) = 2^x is all real numbers (-∞, ∞), and the range is (0, ∞) since the function never touches the x-axis.

How do you evaluate log_10(1000)?

log_10(1000) evaluates to 3 because 10^3 = 1000.

What are exponential growth and decay, and how are they represented mathematically?

Exponential growth is represented by the equation f(t) = a e^(kt), where k > 0, and exponential decay is represented by f(t) = a e^(-kt), where k > 0, with 'a' being the initial amount and 't' being time.

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Functions

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Testy matematyczne
Testy dla uczniów i nie tylko. Sprawdź swoją wiedzę matematyczną.

Exercices corrigés - Calcul exact d'intégrales
Déterminer toutes les primitives des fonctions suivantes, sur un intervalle bien choisi : $$\begin {array} {lll} \displaystyle f_1 (x)=5x^3-3x+7&\displaystyle f_2 (x ...

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Exercices corrigés - Intégrales curvilignes
On pourra d'abord montrer que la forme différentielle est fermée, et utiliser le théorème de Poincaré. Pour la recherche des primitives, on résoudra successivement les équations aux dérivées partielles.

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Exercices corrigés - Équations différentielles linéaires du premier ordre - résolution, applications

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