Limit And Continuity Problems With Solution

EXERCISE 9.4

Evaluate the following limits

1. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x}$

Solution : $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{7}$

Put $\frac{1}{x} = t,$

when $x \to \infty$ means $\frac{1}{x} \to 0$

$\therefore \frac{1}{x} \to 0$ means $t \to 0$

$\left[\lim_{t\to 0}(1+t)^{\frac{1}{t}}\right]^{7} = e^{7}$ $\qquad \left[\because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$

$\therefore \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{7x} = e^{7}$

2. $\lim_{x\to 0}(1+x)^{\frac{1}{3x}}$

Solution : $\lim_{x\to 0}(1+x)^{\frac{1}{3x}} = \left[\lim_{x\to 0}(1+x)^{\frac{1}{x}}\right]^{\frac{1}{3}} = e^{\frac{1}{3}}$

$\left[\because \lim_{x\to 0}(1+x)^{\frac{1}{x}} = e\right]$

3. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{\frac{m}{x}}$

Solution : $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{\frac{m}{x}}$

Put $\frac{1}{x} = t$

$\therefore \lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{\frac{m}{x}} = \lim_{\frac{1}{x}\to 0}(1+kt)^{mt}$

Limit and continuity problems are fundamental concepts in calculus that form the bedrock of mathematical analysis and function behavior. Understanding these concepts is crucial for students and professionals alike, as they pave the way for advanced topics in mathematics, physics, engineering, and other fields. This article will explore various limit and continuity problems, providing detailed explanations, examples, and solutions to enhance comprehension.

Understanding Limits

Limits help us understand the behavior of functions as they approach a specific point. The limit of a function $f(x)$ as x approaches a number c is denoted as:

$$
\lim_{x \to c} f(x)
$$

This notation indicates the value that $f(x)$ approaches as x gets arbitrarily close to c.

Types of Limits

1. One-Sided Limits:
- Left-Hand Limit: Denoted as $\lim_{x \to c^-} f(x)$, this represents the limit as x approaches c from the left.
- Right-Hand Limit: Denoted as $\lim_{x \to c^+} f(x)$, this represents the limit as x approaches c from the right.

2. Infinite Limits:
- These occur when the function approaches infinity as x approaches a certain value. For example, if $f(x)$ becomes arbitrarily large as x approaches c, we write $\lim_{x \to c} f(x) = \infty$.

3. Limits at Infinity:
- These limits analyze the behavior of functions as x approaches infinity or negative infinity. For example, $\lim_{x \to \infty} f(x)$.

Limit Problem Example

Problem 1: Find the limit:

$$
\lim_{x \to 2} (3x^2 - 12)
$$

Solution:

1. Substitute $x = 2$ into the function:
$$
3(2^2) - 12 = 3(4) - 12 = 12 - 12 = 0
$$

\]

2. Therefore, the limit is:
$$
\lim_{x \to 2} (3x^2 - 12) = 0
$$

Continuity of Functions

A function $f(x)$ is said to be continuous at a point c if the following three conditions are met:

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

If a function fails to meet any of these conditions, it is discontinuous at that point.

Types of Discontinuities

1. Point Discontinuity: Occurs when $f(c)$ is not defined or the limit does not equal $f(c)$.
2. Jump Discontinuity: Happens when the left-hand and right-hand limits exist but are not equal.
3. Infinite Discontinuity: Occurs when the function approaches infinity at some point.

Continuity Problem Example

Problem 2: Determine if the function $f(x) = \frac{x^2 - 4}{x - 2}$ is continuous at $x = 2$.

Solution:

1. First, evaluate $f(2)$:
$$
f(2) = \frac{2^2 - 4}{2 - 2} = \frac{0}{0} \quad \text{(undefined)}
$$

Since $f(2)$ is undefined, the function is not continuous at $x = 2$.

2. To investigate further, find the limit as x approaches 2:
$$
\lim_{x \to 2} \frac{x^2 - 4}{x - 2}
$$
Factor the numerator:
$$
\lim_{x \to 2} \frac{(x - 2)(x + 2)}{x - 2}
$$
Cancel $(x - 2)$:

$$
\lim_{x \to 2} (x + 2) = 4
$$

3. Since the limit exists but is not equal to $f(2)$, the function is discontinuous at $x = 2$.

Advanced Limit Problems

Let’s delve into a more complex limit problem involving L'Hôpital's Rule, which is useful for evaluating limits that result in indeterminate forms like $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hôpital's Rule Example

Problem 3: Evaluate the limit:

$$
\lim_{x \to 0} \frac{\sin x}{x}
$$

Solution:

1. Direct substitution gives us $\frac{0}{0}$, an indeterminate form.
2. Apply L'Hôpital's Rule:
- Differentiate the numerator and denominator:
$$
\lim_{x \to 0} \frac{\cos x}{1}
$$
3. Substitute $x = 0$:
$$
\frac{\cos(0)}{1} = 1
$$

4. Therefore, the limit is:
$$
\lim_{x \to 0} \frac{\sin x}{x} = 1
$$

Combining Limits and Continuity

Understanding the relationship between limits and continuity can aid in solving more complicated problems. For instance, if a function is continuous over an interval, it ensures that the limits at every point within that interval also exist.

Combined Problem Example

Problem 4: Determine the continuity of $g(x) = \frac{x^2 - 1}{x - 1}$ at $x = 1$ and find the limit.

Solution:

1. Evaluate $g(1)$:
$$
g(1) = \frac{1^2 - 1}{1 - 1} = \frac{0}{0} \quad \text{(undefined)}
$$

2. Find the limit as x approaches 1:
$$
\lim_{x \to 1} \frac{x^2 - 1}{x - 1}
$$
Factor the numerator:
$$
\lim_{x \to 1} \frac{(x - 1)(x + 1)}{x - 1}
$$
Cancel $(x - 1)$:
$$
\lim_{x \to 1} (x + 1) = 2
$$

3. Although the limit exists, $g(1)$ is undefined, indicating a point discontinuity at $x = 1$.

Conclusion

In summary, limit and continuity problems are essential in understanding the behavior of functions. By mastering the concepts of limits, one can analyze function behavior at specific points, while understanding continuity helps identify where functions are well-defined. The examples provided illustrate various techniques to tackle these problems, from basic evaluations to applying L'Hôpital's Rule. Mastery in these areas is crucial for success in calculus and higher-level mathematics.

Frequently Asked Questions

What is the definition of continuity at a point?

A function f(x) is continuous at a point c if the following three conditions are met: 1) f(c) is defined, 2) the limit of f(x) as x approaches c exists, and 3) the limit of f(x) as x approaches c equals f(c).

How do you evaluate the limit of f(x) = (2x^2 - 8)/(x - 4) as x

approaches 4?

First, substitute x = 4 into the function: f(4) = (2(4^2) - 8)/(4 - 4) = 0/0, which is indeterminate. To resolve this, factor the numerator: f(x) = 2(x^2 - 4)/(x - 4) = 2(x + 4)(x - 4)/(x - 4). Cancel (x - 4): f(x) = 2(x + 4). Now, take the limit as x approaches 4: limit as x -> 4 of 2(x + 4) = 2(8) = 16.

What is the significance of the epsilon-delta definition in limits?

The epsilon-delta definition is a formal way to define the limit of a function. It states that for every ε > 0, there exists a δ > 0 such that if |x - c| < δ, then |f(x) - L| < ε. This means that as x gets arbitrarily close to c, f(x) gets arbitrarily close to L.

Can you provide an example of a function that is continuous everywhere but not differentiable at one point?

Yes, the absolute value function f(x) = |x| is continuous everywhere but not differentiable at x = 0. At this point, the left-hand derivative is -1 and the right-hand derivative is 1, indicating a cusp.

How do you determine if the limit of a piecewise function exists at the boundary point?

To determine if the limit of a piecewise function exists at a boundary point c, evaluate the left-hand limit (as x approaches c from the left) and the right-hand limit (as x approaches c from the right). If both limits exist and are equal, then the limit at that point exists.

Limit And Continuity Problems With Solution

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