

# Gauss Law Practice Problems

## Gauss's Law Practice Worksheet

[http://bohr.physics.arizona.edu/~leone/ua\\_spring\\_2009/phys241lab.html](http://bohr.physics.arizona.edu/~leone/ua_spring_2009/phys241lab.html)

Student Name: \_\_\_\_\_

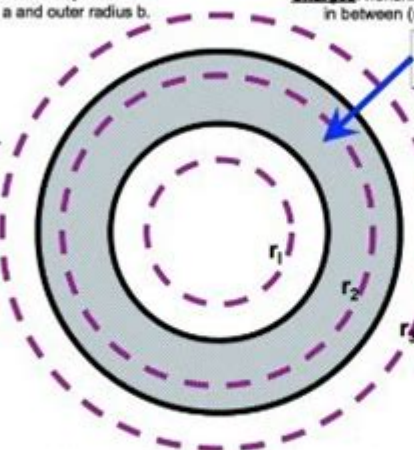
1. **Gauss's Law applied to systems with spherical symmetry:** A Cartesian representation of the electric vector field,  $\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$ , is useless. Try using a spherical coordinate system representation,  $\vec{E} = E_r\hat{r} + E_\theta\hat{\theta} + E_\phi\hat{\phi}$ . Because of the symmetry of the system,  $E_\theta = 0$  and  $E_\phi = 0$ . So  $\vec{E} = E_r\hat{r}$ . Thus the problem of solving for a vector field reduces to a problem of solving for a scalar field quantity that only depends on the radial distance,  $E_r$ .

**System:** Charged hollow sphere with inner radius  $a$  and outer radius  $b$ .

**Charges:** nonuniform charge distribution in between (so not a conductor):

$$\rho(r) = \frac{A}{r}$$

**Problem:** The electric field is a radial vector field due to the symmetry of the system. Find the electric field magnitude in the radial direction at every distance from the origin.



**Required vector calculus knowledge:**

$$\begin{aligned} \int_{\text{charge}} dQ &= \int_{\text{volume}} (\rho) dV \\ &= \int_0^{2\pi} d\phi \int_0^\pi \sin\theta d\theta \int_a^b (\rho) r^2 dr \\ &= 4\pi \int_a^b \left(\frac{A}{r}\right) r^2 dr \\ &= 4\pi \int_a^b A r dr \end{aligned}$$

**Problem solving strategy:** 1) Draw non-physical Gaussian sphere at distance  $r$  where you want to find  $E_r$ . 2) Use Gauss's law to write equation for  $E_r$  in terms of other parameters. 3) Solve for  $E_r$ . In this case solve in 3 places, inside hollow region ( $r_1$ ), inside charged region ( $r_2$ ) and outside ( $r_3$ ).

- Is the electric field constant in magnitude for a fixed radius? Explain.
- In which direction does the electric field point, and how does this depend on the sign of  $\rho$ ? Explain.
- Explain why the sphere is the appropriate Gaussian shape to draw?
- What must the units of the constant  $A$  be?

**Gauss law practice problems** are an essential aspect of mastering electrostatics in physics. Gauss's Law, one of the four Maxwell's equations, relates the electric field over a closed surface to the charge enclosed by that surface. Understanding and practicing Gauss's Law can not only help students excel in their coursework but also prepare them for real-world applications in electrical engineering, physics research, and various technological advancements. This article will delve into Gauss's Law, provide a step-by-step guide to solving practice problems, and present a variety of example problems with solutions.

## Understanding Gauss's Law

Gauss's Law states that the electric flux,  $\Phi_E$ , through a closed surface is

proportional to the charge  $(Q_{\text{enc}})$  enclosed within that surface. Mathematically, it is expressed as:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

Where:

- $(\Phi_E)$  is the electric flux through the closed surface,
- $(\vec{E})$  is the electric field,
- $(d\vec{A})$  is the differential area vector,
- $(Q_{\text{enc}})$  is the charge enclosed by the surface, and
- $(\epsilon_0)$  is the permittivity of free space  $(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)$ .

## Applications of Gauss's Law

Gauss's Law is particularly useful in calculating electric fields for symmetric charge distributions. The following are common applications:

- Point charges
- Spherical charge distributions
- Cylindrical charge distributions
- Planar charge distributions

Each of these configurations allows for straightforward calculations of electric fields using symmetry, simplifying the analysis significantly.

## Steps to Solve Gauss's Law Problems

To effectively solve problems using Gauss's Law, follow these steps:

1. **Identify the symmetry:** Determine the symmetry of the charge distribution (spherical, cylindrical, or planar).
2. **Choose an appropriate Gaussian surface:** Select a closed surface that matches the symmetry of the charge distribution.
3. **Calculate the electric field:** Use Gauss's Law to relate the electric field to the enclosed charge.

4. **Evaluate the integral:** Depending on the symmetry, evaluate the integral to find the electric field.
5. **Check your work:** Verify that your answer makes sense dimensionally and contextually.

## Example Gauss Law Practice Problems

### Problem 1: Electric Field Due to a Point Charge

Problem Statement: Calculate the electric field at a distance  $(r)$  from a point charge  $(Q = 5 \text{ } \mu\text{C})$ .

Solution:

1. Identify the symmetry: The point charge has spherical symmetry.
2. Choose a Gaussian surface: A sphere of radius  $(r)$  centered at the point charge.
3. Calculate the electric field:
  - The total charge enclosed,  $(Q_{\text{enc}} = 5 \text{ } \mu\text{C} = 5 \times 10^{-6} \text{ C})$ .
  - The electric flux through the Gaussian surface:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi r^2$$

- According to Gauss's Law:

$$E \cdot 4\pi r^2 = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$E = \frac{Q_{\text{enc}}}{4\pi \epsilon_0 r^2}$$

4. Substitute values:

$$E = \frac{5 \times 10^{-6}}{4\pi (8.85 \times 10^{-12}) r^2}$$

This gives the electric field due to a point charge.

### Problem 2: Electric Field of an Infinite Plane Sheet of Charge

Problem Statement: Calculate the electric field produced by an infinite plane sheet with surface charge density  $(\sigma = 2 \text{ } \mu\text{C/m}^2)$ .

Solution:

1. Identify the symmetry: The charge distribution is planar and uniform.
2. Choose a Gaussian surface: A cylindrical Gaussian surface (pillbox) with end caps perpendicular to the sheet.
3. Calculate the electric field:

- The charge enclosed by the pillbox is:

$$Q_{\text{enc}} = \sigma A$$

- The electric flux:

$$\Phi_E = E \cdot A + E \cdot A = 2EA$$

- According to Gauss's Law:

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

4. Substitute values:

$$E = \frac{2 \times 10^{-6}}{2(8.85 \times 10^{-12})}$$

This will yield the electric field magnitude due to the infinite plane sheet.

## Problem 3: Electric Field Inside and Outside a Charged Spherical Shell

Problem Statement: A uniformly charged spherical shell has a total charge  $(Q = 10 \, \mu\text{C})$  and a radius  $(R = 0.5 \, \text{m})$ . Determine the electric field at a point  $(r = 0.3 \, \text{m})$  (inside) and  $(r = 0.7 \, \text{m})$  (outside) the shell.

Solution:

1. For  $(r = 0.3 \, \text{m})$  (inside):

- No charge is enclosed within the shell.

- According to Gauss's Law:

$$E \cdot 4\pi(0.3)^2 = 0$$

$$E = 0$$

2. For  $(r = 0.7 \, \text{m})$  (outside):

- The entire charge  $(Q)$  is enclosed.

- The electric flux:

$$\Phi_E = E \cdot 4\pi(0.7)^2$$

\]

- According to Gauss's Law:

\[

$$E \cdot 4\pi(0.7)^2 = \frac{Q}{\epsilon_0}$$

\]

\[

$$E = \frac{Q}{4\pi \epsilon_0 (0.7)^2}$$

\]

3. Substituting values:

\[

$$E = \frac{10 \times 10^{-6}}{4\pi(8.85 \times 10^{-12})(0.7)^2}$$

\]

This approach clearly distinguishes the behavior of electric fields both inside and outside charged spherical shells.

## Conclusion

Mastering **Gauss law practice problems** is crucial for students and professionals in physics and engineering. By understanding the fundamental principles behind Gauss's Law and practicing various problems, you can develop a strong foundation in electrostatics. The problems discussed in this article illustrate the application of Gauss's Law across different charge distributions and geometries. Regular practice will not only improve your problem-solving skills but also enhance your understanding of electric fields and their behavior in various physical scenarios.

## Frequently Asked Questions

### What is Gauss's Law and how is it applied in electrostatics?

Gauss's Law states that the electric flux through a closed surface is proportional to the charge enclosed within that surface. It is applied in electrostatics to simplify the calculation of electric fields for symmetric charge distributions.

### How do you apply Gauss's Law to find the electric field of a uniformly charged spherical shell?

To find the electric field using Gauss's Law for a uniformly charged spherical shell, you consider a Gaussian surface outside the shell. The electric field is zero inside the shell and at a distance 'r' from the center (where r is greater than the radius of the shell), the electric field is  $E = kQ/r^2$ , where Q is the total charge and k is Coulomb's constant.

## Can Gauss's Law be used for non-uniform charge distributions?

Gauss's Law can be used for non-uniform charge distributions, but it is most effective when the symmetry of the system allows for simplifications. For complex distributions, it may be necessary to use calculus to determine the electric field.

## What is the significance of the Gaussian surface choice in Gauss's Law problems?

The choice of Gaussian surface is crucial in Gauss's Law problems because it determines how easily you can calculate the electric flux. It should be chosen to exploit symmetry in the charge distribution to simplify the calculations.

## How does Gauss's Law relate to the concept of electric field lines?

Gauss's Law is related to electric field lines in that the number of electric field lines crossing a surface is proportional to the charge enclosed. A dense concentration of field lines indicates a strong electric field, consistent with higher charge densities.

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## Gauss Law Practice Problems

Support Vector

Machine, SVM Radial Basis Function, RBF Principal ...

gaussian -

Origin Area version of Gaussian Function Gaussian FWHM (the Full Width at Half Maximum) version of Gaussian Function ...

Jacobi Gauss-Seidel

Gauss-Seidel Gauss-Seidel Jacobi Gauss-Seidel ...

origin FWHM

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gaussian

Gaussian Gaussian

GAUSS\_SCRDIR GAUSS\_EXEDIR

*LU* Gauss -

LU LU LU0 LU LU Gauss ...

-

Gauss-Newton Levenberg-Marquardt Hessian Gauss-Newton ...

**Gauss** -

Gauss

4 *Einstein-Gauss-Bonnet*

Gauss-Bonnet Ricci  $R^2$  [1] Gauss-Bonnet Shockwave Solution ...

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Rabbe Bertrand Gauss Bertrand ...

? -

Radial Basis Function, RBF Support Vector Machine, SVM Principal Component ...

*origin* *gauss* *gaussian* -

Origin Gauss Area version of Gaussian Function Guassian FWHM (the Full Width at Half Maximum) version of Gaussian Function

*Jacobi* *Gauss-Seidel* -

Gauss-Seidel Gauss-Seidel Jacobi Gauss-Seidel ...

*origin* *FWHM* -

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*gaussian*

Gaussian Gaussian GAUSS\_SCRDIR GAUSS\_EXEDIR ...

*LU* Gauss -

LU LU LU0 LU LU Gauss ...

-

Gauss-Newton Levenberg-Marquardt Hessian Gauss-Newton ...

**Gauss** -

Gauss

4 *Einstein-Gauss-Bonnet*

Gauss-Bonnet  $\int \text{Ricci} \, dV = 2\pi \chi(M)$  [1] Gauss-Bonnet  $\int \text{Ricci} \, dV = 2\pi \chi(M)$   
Shockwave Solution ...

$\int \text{Ricci} \, dV = 2\pi \chi(M)$  -  $\int \text{Ricci} \, dV = 2\pi \chi(M)$   
Rabbe  $\int \text{Ricci} \, dV = 2\pi \chi(M)$  Bertrand  $\int \text{Ricci} \, dV = 2\pi \chi(M)$  Gauss  $\int \text{Ricci} \, dV = 2\pi \chi(M)$  Bertrand  $\int \text{Ricci} \, dV = 2\pi \chi(M)$  ...

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