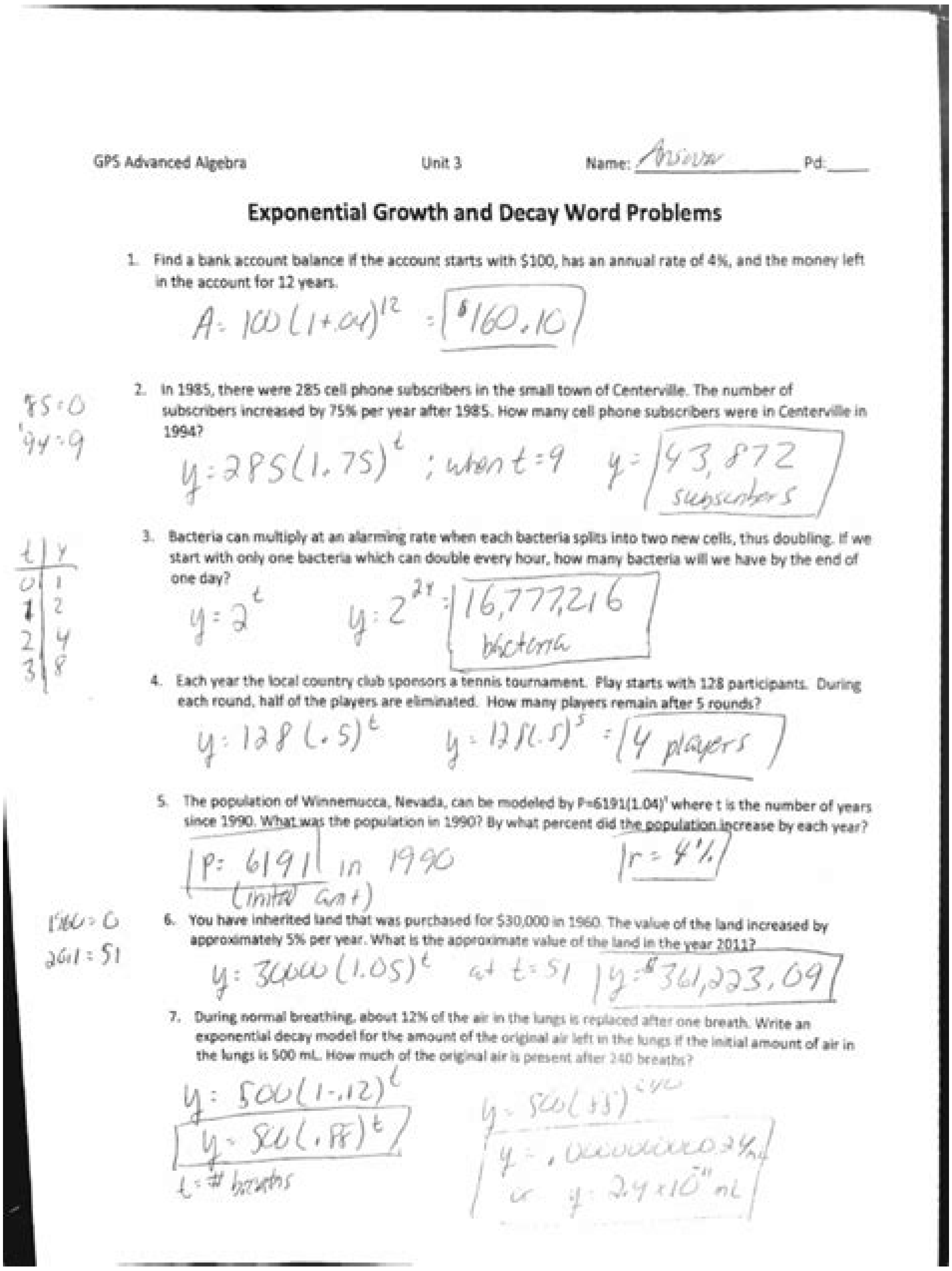
Exponential Growth And Decay Word Problems Answer Key



GPS Advanced Algebra Unit 3 Name: Answer Pd:____

Exponential Growth and Decay Word Problems

1. Find a bank account balance if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

   $A = 100(1+.04)^{12} = \$160.10$

2. In 1985, there were 285 cell phone subscribers in the small town of Centerville. The number of subscribers increased by 75% per year after 1985. How many cell phone subscribers were in Centerville in 1994?

   $y = 285(1.75)^t$; when $t = 9$ $y = 43,872$ subscribers

3. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If we start with only one bacteria which can double every hour, how many bacteria will we have by the end of one day?

   $y = 2^t$ $y = 2^{24} = 16,777,216$ bacteria

4. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

   $y = 128(.5)^t$ $y = 128(.5)^5 = 4$ players

5. The population of Winnemucca, Nevada, can be modeled by $P=6191(1.04)^t$ where t is the number of years since 1990. What was the population in 1990? By what percent did the population increase by each year?

   $P = 6191$ in 1990 (initial amt) $r = 4\%$

6. You have inherited land that was purchased for $30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

   $y = 30000(1.05)^t$ at $t = 51$ $y = \$361,223.09$

7. During normal breathing, about 12% of the air in the lungs is replaced after one breath. Write an exponential decay model for the amount of the original air left in the lungs if the initial amount of air in the lungs is 500 mL. How much of the original air is present after 240 breaths?

   $y = 500(1-.12)^t$ $y = 500(.88)^t$ t = # breaths

   $y = 500(.88)^{240}$ $y = .0000000024$ mL or $y = 2.4 \times 10^{-11}$ mL

Exponential growth and decay word problems answer key are essential for understanding how various phenomena change over time in real-life situations. Exponential functions are pivotal in fields such as biology, finance, and physics. This article will delve into the concept of exponential growth and decay, provide various examples of word problems, and present an answer key to help clarify the solutions to these problems. By grasping these concepts, students can strengthen their problem-solving skills and apply their understanding in practical contexts.

Understanding Exponential Growth and Decay

Exponential growth and decay describe processes that change at a rate proportional to their current value. In mathematical terms, these processes can be modeled using the exponential function:

- Exponential Growth: This occurs when a quantity increases rapidly at a rate that is proportional to its current value. The general formula for exponential growth is:

$$
N(t) = N_0 e^{kt}
$$

where:
- $N(t)$ is the quantity at time t,
- N_0 is the initial quantity,
- k is the growth constant,
- e is the base of the natural logarithm (approximately equal to 2.71828).

- Exponential Decay: This occurs when a quantity decreases rapidly at a rate that is proportional to its current value. The general formula for exponential decay is:

$$
N(t) = N_0 e^{-kt}
$$

where the variables are defined similarly to those in the growth formula.

Key Characteristics

1. Growth Factor: In exponential growth, the quantity increases by a fixed percentage over equal time intervals.
2. Decay Factor: In exponential decay, the quantity decreases by a fixed percentage over equal time intervals.
3. Time Constant: The time it takes for a quantity to decrease to about 37% of its original value in decay models.

Applications of Exponential Growth and Decay

Exponential functions are used in various fields, including:

- Biology: Modeling population growth.
- Finance: Calculating compound interest.
- Physics: Radioactive decay.
- Medicine: Drug decay in the body.

Exponential Growth and Decay Word Problems

Below are several examples of word problems that illustrate exponential growth and decay, followed by their solutions.

Example 1: Population Growth

A certain city has a population of 50,000 people. The population is expected to grow at a rate of 3% per year. What will the population be in 10 years?

Solution:
Using the exponential growth formula:

- $N_0 = 50000$
- $k = 0.03$ (3% as a decimal)
- $t = 10$

Plugging in the values:

$$
N(10) = 50000 e^{0.03 \times 10}
$$

Calculating:

$$
N(10) = 50000 e^{0.3} \approx 50000 \times 1.34986 \approx 67493
$$

So, the population will be approximately 67,493 in 10 years.

Example 2: Radioactive Decay

A particular radioactive substance has a half-life of 5 years. If you start with 80 grams of the substance, how much will remain after 15 years?

Solution:
The amount remaining after t years can be calculated using the formula:

$$
N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}
$$

where h is the half-life.

- $N_0 = 80$ grams
- $h = 5$ years
- $t = 15$ years

Plugging in the values:

$$
N(15) = 80 \left(\frac{1}{2}\right)^{\frac{15}{5}} = 80 \left(\frac{1}{2}\right)^{3} = 80 \times \frac{1}{8} = 10
$$

After 15 years, 10 grams of the substance will remain.

Example 3: Investment Growth

You invest $1,000 in an account that offers an annual interest rate of 5%, compounded continuously. How much money will be in the account after 20 years?

Solution:
Using the formula for continuous compounding:

$$
A = N_0 e^{rt}
$$

where:
- $N_0 = 1000$

- $r = 0.05$
- $t = 20$

Plugging in the values:

$$
A = 1000 e^{0.05 \times 20} = 1000 e^{1} \approx 1000 \times 2.71828 \approx 2718.28
$$

After 20 years, the account will have approximately $2,718.28.

Example 4: Bacteria Growth

A scientist observes that the number of bacteria in a culture doubles every 3 hours. If there are initially 1,000 bacteria, how many will there be after 12 hours?

Solution:
Since the bacteria doubles every 3 hours, we can use the formula:

$$
N(t) = N_0 \cdot 2^{\frac{t}{d}}
$$

where d is the doubling time.

- $N_0 = 1000$
- $d = 3$ hours
- $t = 12$ hours

Plugging in the values:

$$
N(12) = 1000 \cdot 2^{\frac{12}{3}} = 1000 \cdot 2^{4} = 1000 \cdot 16 = 16000
$$

After 12 hours, there will be 16,000 bacteria.

Answer Key Summary

1. Population Growth: 67,493

2. Radioactive Decay: 10 grams
3. Investment Growth: $2,718.28
4. Bacteria Growth: 16,000

Conclusion

Understanding exponential growth and decay is vital for solving real-world problems across various disciplines. By practicing with different types of word problems, students can gain confidence and proficiency in applying these concepts. The answer key provided serves as a useful reference for verifying solutions and clarifying misunderstandings. As you encounter more complex problems, continue to apply these foundational principles to enhance your analytical skills and deepen your understanding of exponential phenomena.

Frequently Asked Questions

What is an example of an exponential growth word problem?

A common example is a population growth problem where a city has a population of 50,000 that grows by 5% each year. The formula used would be $P(t) = P0\ e^{(rt)}$, where P0 is the initial population, r is the growth rate, and t is the time in years.

How do you solve an exponential decay problem involving radioactive decay?

To solve a radioactive decay problem, use the formula $N(t) = N0\ e^{(-kt)}$, where N0 is the initial quantity, k is the decay constant, and t is time. For example, if a substance has a half-life of 10 years, you can find the remaining quantity after a certain time by substituting t into the equation.

What is the difference between exponential growth and decay?

Exponential growth refers to a situation where a quantity increases rapidly over time, often modeled by a positive exponent. In contrast, exponential decay describes a scenario where a quantity decreases rapidly, represented by a negative exponent.

How can exponential growth be modeled in a business context?

In a business context, exponential growth can be modeled by looking at sales revenue that increases by a constant percentage each month. The formula used would be $R(t) = R0\ (1 + r)^t$, where R0 is the initial revenue, r is the growth rate, and t is the time in months.

What are some common real-world applications of exponential decay?

Common applications of exponential decay include modeling radioactive decay in physics, calculating depreciation of assets in finance, and analyzing the cooling of objects in thermodynamics.

Exponential Growth And Decay Word Problems Answer Key

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Unlock the secrets of exponential growth and decay with our comprehensive answer key for word problems. Learn more and master your math skills today!

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