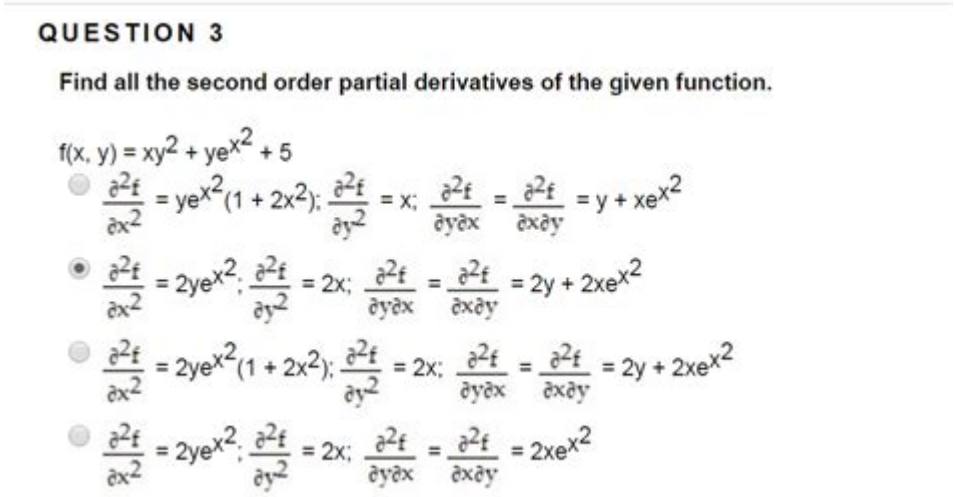
Examples Of Calculus Problems With Answers



QUESTION 3

Find all the second order partial derivatives of the given function.

$f(x, y) = xy^2 + ye^{x^2} + 5$

- $\frac{\partial^2 f}{\partial x^2} = ye^{x^2}(1 + 2x^2); \frac{\partial^2 f}{\partial y^2} = x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = y + xe^{x^2}$
- $\frac{\partial^2 f}{\partial x^2} = 2ye^{x^2}; \frac{\partial^2 f}{\partial y^2} = 2x; \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = 2y + 2xe^{x^2}$
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Examples of calculus problems with answers are essential for students and enthusiasts looking to deepen their understanding of this fundamental branch of mathematics. Calculus, which focuses on the concepts of change and motion, is divided mainly into two parts: differential calculus and integral calculus. This article will present a variety of calculus problems, ranging from basic to advanced levels, along with their solutions. By working through these examples, readers can enhance their problem-solving skills and build a solid foundation in calculus.

1. Basic Differentiation Problems

Differentiation is a key concept in calculus that deals with finding the rate at which a function is changing. Here are some basic differentiation problems:

Problem 1: Differentiate the function

Given the function $f(x) = 3x^2 + 5x - 4$, find $f'(x)$.

Solution:
To differentiate $f(x)$, we apply the power rule:
$$
f'(x) = \frac{d}{dx}(3x^2) + \frac{d}{dx}(5x) - \frac{d}{dx}(4)
$$
Calculating each term, we have:
$$
f'(x) = 6x + 5 - 0 = 6x + 5
$$

Problem 2: Differentiate the trigonometric function

Differentiate $g(x) = \sin(x) + \cos(x)$.

Solution:
Using the derivatives of sine and cosine, we find:
$$
g'(x) = \cos(x) - \sin(x)
$$

2. Intermediate Differentiation Problems

As we progress, differentiation becomes more complex, involving products, quotients, and chain rules.

Problem 3: Product Rule

Differentiate the function $h(x) = x^2 \cdot e^x$.

Solution:
Using the product rule, which states that if $u(x)$ and $v(x)$ are functions, then:
$$
(uv)' = u'v + uv'
$$
Let $u(x) = x^2$ and $v(x) = e^x$. Then:
$$
u' = 2x, \quad v' = e^x
$$
Applying the product rule:
$$
h'(x) = (2x)(e^x) + (x^2)(e^x) = e^x(2x + x^2)
$$

Problem 4: Quotient Rule

Differentiate $k(x) = \frac{x^3 - 1}{x^2 + 1}$.

Solution:
Using the quotient rule, which states that if $u(x)$ and $v(x)$ are functions, then:
$$
\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}
$$
Let $u(x) = x^3 - 1$ and $v(x) = x^2 + 1$. Then:
$$
u' = 3x^2, \quad v' = 2x
$$
Applying the quotient rule:
$$

$$
k'(x) = \frac{(3x^2)(x^2 + 1) - (x^3 - 1)(2x)}{(x^2 + 1)^2} = \frac{3x^4 + 3x^2 - 2x^4 + 2x}{(x^2 + 1)^2} = \frac{x^4 + 3x^2 + 2x}{(x^2 + 1)^2}
$$

3. Basic Integration Problems

Integration is the process of finding the accumulated area under a curve. Here are some basic integration problems:

Problem 5: Indefinite Integral

Evaluate the integral $\int (4x^3 - 2x + 1) \, dx$.

Solution:
Using the power rule for integration:
$$
\int (4x^3) \, dx = x^4, \quad \int (-2x) \, dx = -x^2, \quad \int (1) \, dx = x
$$
Combining these results, we get:
$$
\int (4x^3 - 2x + 1) \, dx = x^4 - x^2 + x + C
$$
where C is the constant of integration.

Problem 6: Definite Integral

Evaluate $\int_0^1 (2x) \, dx$.

Solution:
First, compute the indefinite integral:
$$
\int (2x) \, dx = x^2 + C
$$
Now, apply the limits from 0 to 1:
$$
\left[x^2 \right]_0^1 = 1^2 - 0^2 = 1 - 0 = 1
$$

4. Advanced Problems in Calculus

In this section, we will tackle more complex calculus problems that require deeper understanding and application of concepts.

Problem 7: Applying the Chain Rule

Differentiate the function $j(x) = \sqrt{5x^2 + 3}$.

Solution:
Using the chain rule, where $u = 5x^2 + 3$ and $y = \sqrt{u}$:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot (10x) = \frac{10x}{2\sqrt{5x^2 + 3}} = \frac{5x}{\sqrt{5x^2 + 3}}$$

Problem 8: Evaluating a Limit

Evaluate the limit $\lim_{x \to 0} \frac{\sin(2x)}{x}$.

Solution:
Using L'Hôpital's Rule, which states that if the limit results in $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

$$\lim_{x \to 0} \frac{\sin(2x)}{x} = \lim_{x \to 0} \frac{2\cos(2x)}{1} = 2\cos(0) = 2$$

5. Conclusion

Calculus is a vast and rich field of study that encompasses various concepts such as differentiation, integration, and limits. The examples of calculus problems with answers provided in this article are designed to help learners at different levels enhance their understanding and problem-solving skills. By practicing these problems, students can gain confidence in their ability to tackle calculus challenges and apply these concepts in real-world scenarios. Whether you are a beginner or looking to refine your skills, working through these examples is a valuable step in mastering calculus.

Frequently Asked Questions

What is the derivative of the function f(x) = 3x^2 + 5x - 4?

The derivative f'(x) = 6x + 5.

How do you find the integral of f(x) = 2x with respect to x?

The integral ∫2x dx = x^2 + C, where C is the constant of integration.

What is an example of a limit problem and its solution?

Find the limit as x approaches 2 for f(x) = x^2 - 4. The limit is 0, since f(2) = 0.

Can you provide an example of a related rates problem?

If a balloon is rising at 5 ft/s and its radius is increasing at 2 ft/s, find the rate of change of the volume. The volume V = (4/3)πr^3, and the rate of change is dV/dt = 4πr^2(dr/dt), where dr/dt = 2 ft/s.

What is an example of using the Fundamental Theorem of Calculus?

If F(x) = ∫ from 1 to x (t^2 dt), then F'(x) = x^2.

How do you solve a definite integral like ∫ from 0 to 1 (3x^2 dx)?

The definite integral evaluates to [x^3] from 0 to 1, which equals 1 - 0 = 1.

What is an example of optimization using calculus?

To maximize the area of a rectangle given a fixed perimeter of 20, set up the equation A = lw, subject to the constraint l + w = 10. The maximum area occurs when l = w = 5, giving A = 25.

Can you give an example of finding a critical point?

For f(x) = x^3 - 3x^2 + 4, find f'(x) = 3x^2 - 6. Setting f'(x) = 0 gives critical points at x = 0 and x = 2.

Examples Of Calculus Problems With Answers

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examples - WordReference.com Dictionary of English
a pattern or model, as of something to be imitated or avoided: to set a good example. for instance: The train I take is always late. For example, this morning it was a half an hour late. See -am-.

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An example of something is a particular situation, object, or person that shows that what is being claimed is true. The doctors gave numerous examples of patients being expelled from the hospital.

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Examples - definition of Examples by The Free Dictionary
An example is a typically representative part that demonstrates the character of the whole: "Of the despotism to which unrestrained military power leads we have plenty of examples from Alexander to Mao" (Samuel Eliot Morison).

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