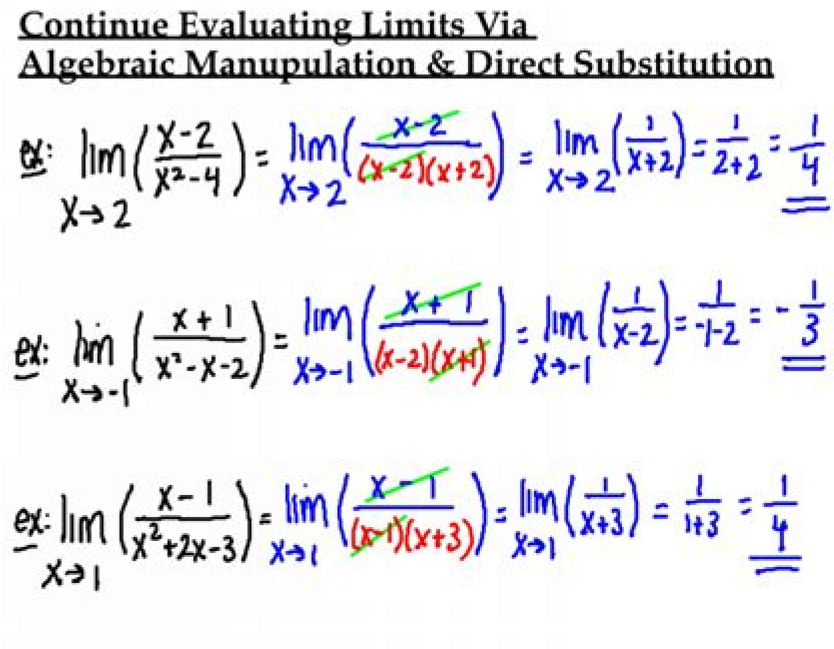
Example Of A Calculus Problem



Example of a calculus problem can take many forms, from basic differentiation to complex integration challenges. In this article, we will explore a specific example that encompasses various concepts in calculus, demonstrating how to approach and solve a problem step by step. This example will not only clarify the methods used in calculus but also illustrate the importance of these techniques in real-world scenarios.

Understanding the Problem

To provide a clear example, let's consider a real-world context: a company that manufactures and sells a product. The company's revenue and cost functions are defined as follows:

- Revenue Function: \(R(x) = 50x - 0.5x^2 \)
- Cost Function: \(C(x) = 20x + 100 \)

Here, \(x \) represents the number of units sold. Our goal is to find the number of units that maximizes the company's profit.

Step 1: Defining Profit

Profit is defined as the difference between revenue and cost. Thus, we can express the profit function \(P(x) \) as follows:

\[
P(x) = R(x) - C(x)
\]

Substituting the given functions into this equation, we have:

\[

$$
P(x) = (50x - 0.5x^2) - (20x + 100)
$$

Simplifying this expression, we can combine like terms:

$$
P(x) = 50x - 0.5x^2 - 20x - 100
$$

$$
P(x) = -0.5x^2 + 30x - 100
$$

Step 2: Finding the Critical Points

To find the maximum profit, we need to locate the critical points of the profit function by taking the derivative and setting it to zero. The derivative $P'(x)$ is calculated as follows:

$$
P'(x) = \frac{d}{dx}(-0.5x^2 + 30x - 100)
$$

$$
P'(x) = -x + 30
$$

Setting the derivative equal to zero gives us:

$$
-x + 30 = 0
$$

Solving for x:

$$
x = 30
$$

Step 3: Determining the Nature of the Critical Point

To confirm that this critical point represents a maximum, we can analyze the second derivative $P''(x)$:

$$
P''(x) = \frac{d^2}{dx^2}(-0.5x^2 + 30x - 100)
$$

$$
P''(x) = -1
$$

Since $P''(x) < 0$, this indicates that the function is concave down at $x = 30$, confirming that this point indeed represents a maximum profit.

Step 4: Calculating Maximum Profit

Now that we have determined the number of units that maximizes profit, we can substitute $x = 30$ back into the profit function to find the maximum profit:

$$
P(30) = -0.5(30^2) + 30(30) - 100
$$
$$
P(30) = -0.5(900) + 900 - 100
$$
$$
P(30) = -450 + 900 - 100
$$
$$
P(30) = 350
$$

Thus, the maximum profit is \$350 when the company sells 30 units.

Graphical Representation

To better understand the relationship between the profit, revenue, and cost functions, we can visualize them on a graph.

Plotting the Functions

1. Revenue Function: $R(x) = 50x - 0.5x^2$
2. Cost Function: $C(x) = 20x + 100$
3. Profit Function: $P(x) = -0.5x^2 + 30x - 100$

The graph would typically show:
- The revenue function as a parabola opening downwards.
- The cost function as a linear line with a positive slope.
- The profit function as a second parabola opening downwards, intersecting the x-axis at points where profit is zero.

Interpreting the Graph

- Intersection Points: The points where the revenue and cost functions intersect represent the break-even points.
- Maximum Point: The vertex of the profit function gives the maximum profit point, which we calculated as $x = 30$.

Real-World Applications

Understanding how to maximize profit is crucial in many business scenarios. Here are some practical applications of this calculus problem:

- Business Strategy: Companies can use similar models to determine pricing strategies and production levels to ensure profitability.
- Resource Allocation: Understanding the profit maximization helps in allocating resources effectively.
- Market Analysis: By analyzing the shapes of the revenue and cost functions, companies can predict how changes in market conditions (like demand or production costs) might affect their profits.

Conclusion

This example of a calculus problem illustrates the powerful tools calculus provides for solving real-world issues such as profit maximization. By defining revenue and cost functions, deriving the profit function, and finding its critical points, we demonstrate a systematic approach to optimization problems. Whether in business, engineering, or the sciences, calculus remains an invaluable resource for decision-making and strategic planning.

Understanding these concepts not only aids in academic pursuits but also equips individuals with the analytical skills necessary for various professional fields. The interplay between theoretical mathematics and practical application underscores the importance of calculus in everyday problem-solving.

Frequently Asked Questions

What is an example of a basic calculus problem involving derivatives?

Find the derivative of the function $f(x) = 3x^2 + 2x - 5$.

Can you provide an example of a calculus problem that involves integration?

Calculate the integral of the function $f(x) = 4x^3$ from $x = 1$ to $x = 3$.

What is an example of a calculus problem using the Fundamental Theorem of Calculus?

Evaluate the definite integral of $f(x) = \sin(x)$ from 0 to π.

How do you set up a calculus problem for finding the area under a curve?

To find the area under the curve $y = x^2$ from $x = 0$ to $x = 2$, you would set up the integral: ∫ (from 0 to 2) $x^2\,dx$.

What is an example of applying the Chain Rule in calculus?

Differentiate the function $f(x) = (2x + 3)^5$ using the Chain Rule.

Can you give an example of a calculus problem that involves limits?

Evaluate the limit: $\lim_{x \to 0} (\sin(x)/x)$.

What is an example of a calculus problem involving related rates?

If a balloon is being inflated, and its radius is increasing at a rate of 2 cm/s, how fast is the volume of the balloon increasing when the radius is 5 cm?

Can you provide an example of a calculus optimization problem?

Find the dimensions of a rectangle with a perimeter of 20 meters that maximizes the area.

What is an example of a calculus problem that requires finding the second derivative?

Given the function $f(x) = x^3 - 6x^2 + 9x$, find the second derivative $f''(x)$.

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