Engineering Mathematics 3 Solved Problems

MATH198 Solution Sheet 3

1. *Check whether the solutions listed below satisfy the differential equations, showing your working. A, B and a are constants.*

(i) $\frac{dy}{dt} = -4y$; $\qquad y = Ae^{-4t}$

An easy one to start with.

$$\begin{aligned} y &= Ae^{-4t} \\ \Rightarrow \frac{dy}{dt} &= -4Ae^{-4t} \\ LHS &= \frac{dy}{dt} = -4Ae^{-4t}, \quad RHS = -4y = -4Ae^{-4t} \quad \Rightarrow LHS = RHS \checkmark \end{aligned}$$

The left-hand side and right-hand side are equal for all t, so the solution given was correct.

(ii) $2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 3$; $\qquad y = Ax + \frac{B}{\sqrt{x}} - 3$

Check:

$$\begin{aligned} y &= Ax + Bx^{-\frac{1}{2}} - 3 \\ y' &= A - \tfrac{1}{2}Bx^{-\frac{3}{2}} \\ y'' &= \tfrac{3}{4}Bx^{-\frac{5}{2}} \end{aligned}$$

$$\Rightarrow \quad 2x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 2x^2\left(\tfrac{3}{4}Bx^{-\frac{5}{2}}\right) + x\left(A - \tfrac{1}{2}Bx^{-\frac{3}{2}}\right) - \left(Ax + Bx^{-\frac{1}{2}} - 3\right)$$
$$= \left(\tfrac{3}{2}B - \tfrac{1}{2}B - B\right)x^{-\frac{1}{2}} + Ax - Ax + 3 = 3 \quad \checkmark$$

(iii) $y\,y'' + (y' - y)\,y' = 0$; $\qquad y = \sqrt{Ae^x + B}$

$$\begin{aligned} y &= (Ae^x + B)^{\frac{1}{2}} \\ y' &= \tfrac{1}{2}(Ae^x + B)^{-\frac{1}{2}}\,Ae^x \qquad \text{(Chain rule, } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}\text{, with } u = Ae^x + B.\text{)} \\ y'' &= -\tfrac{1}{4}(Ae^x + B)^{-\frac{3}{2}}\,A^2e^{2x} + \tfrac{1}{2}(Ae^x + B)^{-\frac{1}{2}}\,Ae^x \qquad \text{(Product rule and chain rule)} \end{aligned}$$

(When you take the second derivative the product rule means that there are *two* terms - some people missed out the second term.)

$$\begin{aligned} y\,y'' + (y' - y)\,y' &= (Ae^x + B)^{\frac{1}{2}}\left[-\tfrac{1}{4}(Ae^x + B)^{-\frac{3}{2}}\,A^2e^{2x} + \tfrac{1}{2}(Ae^x + B)^{-\frac{1}{2}}\,Ae^x\right] \\ &\quad + \left[\tfrac{1}{2}(Ae^x + B)^{-\frac{1}{2}}\,Ae^x - (Ae^x + B)^{\frac{1}{2}}\right]\tfrac{1}{2}(Ae^x + B)^{-\frac{1}{2}}\,Ae^x \\ &= -\tfrac{1}{4}(Ae^x + B)^{-1}A^2e^{2x} + \tfrac{1}{2}Ae^x + \tfrac{1}{4}(Ae^x + B)^{-1}A^2e^{2x} - \tfrac{1}{2}Ae^x \\ &= 0 \quad \checkmark \end{aligned}$$

Engineering mathematics 3 solved problems encompass a wide range of mathematical concepts and techniques that are crucial for engineering students. This branch of mathematics typically covers areas such as differential equations, vector calculus, complex variables, and numerical methods. Mastering these topics not only helps students in their academic pursuits but also provides essential tools for solving practical engineering problems. In this article, we will explore various solved problems in engineering mathematics 3, providing clear explanations and insights into each concept.

Understanding Differential Equations

Differential equations play a fundamental role in modeling various engineering phenomena. They describe the relationship between a function and its derivatives and can be classified into ordinary differential equations (ODEs) and partial differential equations (PDEs).

Example Problem 1: First-Order Linear ODE

Consider the first-order linear ordinary differential equation:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = 2x$ and $Q(x) = e^{-x^2}$.

Solution Steps:

1. Identify $P(x)$ and $Q(x)$:
- $P(x) = 2x$
- $Q(x) = e^{-x^2}$

2. Find the Integrating Factor:
The integrating factor $\mu(x)$ is given by:
$$\mu(x) = e^{\int P(x) \, dx} = e^{\int 2x \, dx} = e^{x^2}$$

3. Multiply the ODE by the Integrating Factor:
$$e^{x^2} \frac{dy}{dx} + e^{x^2} \cdot 2xy = e^{x^2} e^{-x^2}$$
Simplifying gives:
$$e^{x^2} \frac{dy}{dx} + 2xy e^{x^2} = 1$$

4. Rearranging the equation:
$$\frac{d}{dx}(y e^{x^2}) = 1$$

5. Integrate both sides:
$$y e^{x^2} = x + C$$
where C is the constant of integration.

6. Solve for y:
$$y = (x + C)e^{-x^2}$$

Final Solution:
The general solution to the differential equation is:
$$y = (x + C)e^{-x^2}$$

Vector Calculus and Its Applications

Vector calculus is an essential area of mathematics that deals with vector fields and their derivatives. It is particularly useful in physics and engineering, providing the mathematical framework for analyzing phenomena such as fluid flow and electromagnetic fields.

Example Problem 2: Gradient, Divergence, and Curl

Given a scalar field $\phi(x, y, z) = x^2 + y^2 + z^2$ and a vector field $\mathbf{F}(x, y, z) = (xy, yz, zx)$, compute the gradient of ϕ, the divergence of $\mathbf{F}$, and the curl of $\mathbf{F}$.

Solution Steps:

1. Calculate the Gradient of ϕ:
The gradient $\nabla \phi$ is given by:

$$\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right)$$

Therefore:

$$\frac{\partial \phi}{\partial x} = 2x, \quad \frac{\partial \phi}{\partial y} = 2y, \quad \frac{\partial \phi}{\partial z} = 2z$$

Thus,

$$\nabla \phi = (2x, 2y, 2z)$$

2. Calculate the Divergence of $\mathbf{F}$:
The divergence $\nabla \cdot \mathbf{F}$ is given by:

$$\nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Therefore:

$$\frac{\partial (xy)}{\partial x} + \frac{\partial (yz)}{\partial y} + \frac{\partial (zx)}{\partial z} = y + z + x$$

Thus,

$$\nabla \cdot \mathbf{F} = x + y + z$$

3. Calculate the Curl of $\mathbf{F}$:
The curl $\nabla \times \mathbf{F}$ is given by:

$$\nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right)$$

Therefore:

$$\frac{\partial (zx)}{\partial y} - \frac{\partial (yz)}{\partial z} = 0 - y = -y$$

$$
\frac{\partial (xy)}{\partial z} - \frac{\partial (zx)}{\partial x} = 0 - z = -z
$$
$$
\frac{\partial (yz)}{\partial x} - \frac{\partial (xy)}{\partial y} = 0 - x = -x
$$
Thus,
$$
\nabla \times \mathbf{F} = (-y, -z, -x)
$$

Final Results:
- Gradient: $\nabla \phi = (2x, 2y, 2z)$
- Divergence: $\nabla \cdot \mathbf{F} = x + y + z$
- Curl: $\nabla \times \mathbf{F} = (-y, -z, -x)$

Complex Variables in Engineering

Complex variables are another vital area in engineering mathematics, particularly in fields like electrical engineering and fluid dynamics. They provide powerful methods for solving problems that involve oscillatory behavior.

Example Problem 3: Analytic Function

Determine if the function $f(z) = z^2 + 3z + 2$ is analytic in the complex plane.

Solution Steps:

1. Check for Analyticity:
A function $f(z)$ is analytic if it is differentiable in an open region of the complex plane.

2. Calculate the Derivative:
The derivative $f'(z)$ is:
$$
f'(z) = 2z + 3
$$

3. Check the Cauchy-Riemann Equations:
Let $z = x + iy$ where $f(z) = u(x, y) + iv(x, y)$:
- The real part $u(x, y) = x^2 - y^2 + 3x - 3y + 2$
- The imaginary part $v(x, y) = 2xy + 3y$

The Cauchy-Riemann equations are:
$$
\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad
\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}
$$

Calculate the partial derivatives:
- $\frac{\partial u}{\partial x} = 2x + 3$

- $\frac{\partial v}{\partial y} = 2x + 3$
- $\frac{\partial u}{\partial y} = -2y$
- $\frac{\partial v}{\partial x} = 2y$

Both equations hold true, indicating $f(z)$ satisfies the Cauchy-Riemann equations.

Final Conclusion:
The function $f(z) = z^2 + 3z + 2$ is analytic everywhere in the complex plane.

Numerical Methods in Engineering

Numerical methods are essential for approximating solutions to problems that cannot be solved analytically. They are widely used in engineering for simulations, optimizations, and modeling.

Example Problem 4: Newton-Raphson Method

Use the Newton-Raphson method to find a root of the equation $f(x) = x^2 - 4$.

Frequently Asked Questions

What are some common topics covered in Engineering Mathematics 3?

Common topics include differential equations, vector calculus, complex variables, and numerical methods.

Can you provide an example of a solved problem involving differential equations?

Sure! A typical problem would be solving the first-order linear differential equation dy/dx + P(x)y = Q(x) using an integrating factor.

What numerical methods are frequently taught in Engineering Mathematics 3?

Numerical methods such as Euler's method, Runge-Kutta methods, and finite difference methods for solving differential equations are commonly taught.

How is vector calculus applied in engineering mathematics?

Vector calculus is used to analyze and solve problems involving vector fields, such as fluid flow and electromagnetism.

What is the importance of complex variables in engineering mathematics?

Complex variables are important for simplifying problems in electrical engineering, control systems, and fluid dynamics.

How do you solve a partial differential equation (PDE) using separation of variables?

To solve a PDE using separation of variables, assume a solution can be written as a product of functions, each depending on a single variable, and then separate the variables to solve.

Can you explain a solved problem involving Fourier series?

A common problem involves finding the Fourier series expansion of a periodic function, which allows for analysis of signals in frequency domain.

What role does Laplace Transform play in engineering mathematics?

Laplace Transform is used to convert differential equations into algebraic equations, making them easier to solve, especially in control systems.

What types of problems can be solved using linear algebra in engineering?

Linear algebra can be used to solve systems of linear equations, perform transformations, and analyze stability in engineering systems.

How do you apply the method of characteristics to solve hyperbolic PDEs?

The method of characteristics involves transforming the PDE into a set of ordinary differential equations along characteristic curves, which can then be solved for the solution.

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