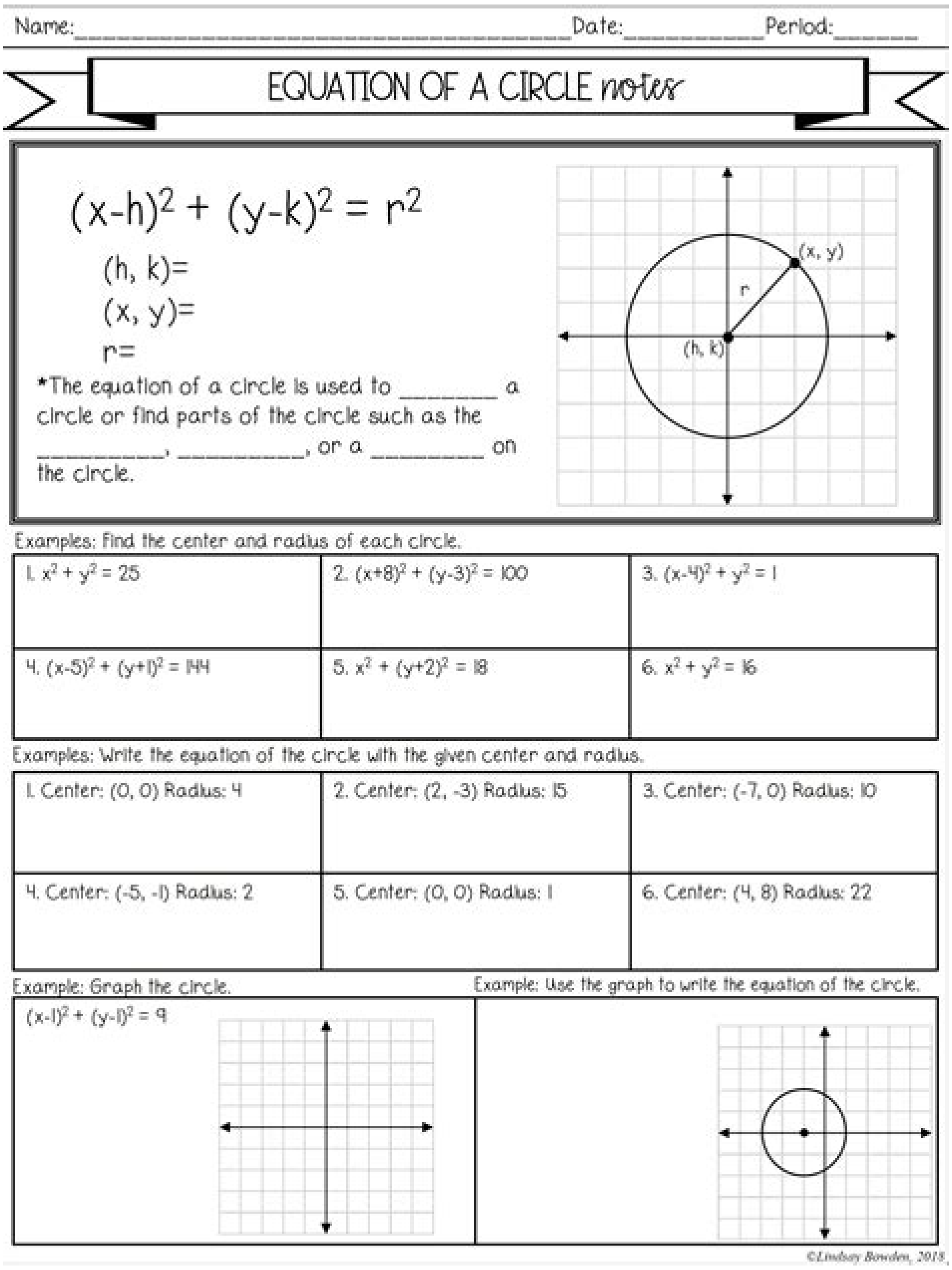
Equation Of A Circle Notes Answer Key



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EQUATION OF A CIRCLE *notes*

$(x-h)^2 + (y-k)^2 = r^2$

(h, k)=

(x, y)=

r=

*The equation of a circle is used to ________ a circle or find parts of the circle such as the __________, __________, or a _________ on the circle.

Examples: Find the center and radius of each circle.

1. $x^2 + y^2 = 25$	2. $(x+8)^2 + (y-3)^2 = 100$	3. $(x-4)^2 + y^2 = 1$
4. $(x-5)^2 + (y+1)^2 = 144$	5. $x^2 + (y+2)^2 = 18$	6. $x^2 + y^2 = 16$

Examples: Write the equation of the circle with the given center and radius.

1. Center: (0, 0) Radius: 4	2. Center: (2, -3) Radius: 15	3. Center: (-7, 0) Radius: 10
4. Center: (-5, -1) Radius: 2	5. Center: (0, 0) Radius: 1	6. Center: (4, 8) Radius: 22

Example: Graph the circle.

$(x-1)^2 + (y-1)^2 = 9$

Example: Use the graph to write the equation of the circle.

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The equation of a circle is a fundamental concept in geometry and algebra. Understanding this equation is crucial for solving various problems in mathematics, physics, and engineering. This article will provide a comprehensive overview of the equation of a circle, including its derivation, standard form, general form, properties, and applications. Additionally, we will include notes and an answer key for better understanding.

1. Definition of a Circle

A circle is defined as the set of all points in a plane that are equidistant from a fixed point known as the center. The distance from the center to any point on the circle is called the radius.

2. Deriving the Equation of a Circle

To derive the equation of a circle, we start with the following:

- Let the center of the circle be at the point (h, k).
- Let the radius of the circle be r.

Using the distance formula, the distance d from the center (h, k) to any point (x, y) on the circle is given by:

$$ d = \sqrt{(x - h)^2 + (y - k)^2} $$

Since all points on the circle are at a distance r from the center, we set $d = r$:

$$ \sqrt{(x - h)^2 + (y - k)^2} = r $$

By squaring both sides, we eliminate the square root:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

This is the standard form of the equation of a circle.

3. Standard Form of the Equation of a Circle

The standard form of the equation of a circle is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

Where:
- (h, k) is the center of the circle.
- r is the radius.

3.1 Identifying Circle Parameters

To identify the center and radius from the standard form:

- Center: The coordinates (h, k)
- Radius: The value r

Example:
For the equation $(x - 3)^2 + (y + 2)^2 = 16$:
- Center: $(3, -2)$
- Radius: $r = \sqrt{16} = 4$

4. General Form of the Equation of a Circle

The general form of the equation of a circle is obtained by expanding the standard form:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

Where D, E, and F are constants derived from the parameters h, k, and r:

- \(D = -2h\)
- \(E = -2k\)
- \(F = h^2 + k^2 - r^2\)

Example:
Starting from \((x - 3)^2 + (y + 2)^2 = 16\):

1. Expand:
- \((x^2 - 6x + 9) + (y^2 + 4y + 4) = 16 \)
2. Combine and rearrange:
- \(x^2 + y^2 - 6x + 4y - 3 = 0 \)

Thus, the general form is:
\[x^2 + y^2 - 6x + 4y - 3 = 0 \]

5. Properties of a Circle

Understanding the properties of a circle is essential for solving related problems. Here are some key properties:

- Symmetry: A circle is symmetric about its center. It has rotational symmetry of order infinite.
- Diameter: The diameter \(d\) is twice the radius: \(d = 2r\).
- Chords: A chord is a line segment with both endpoints on the circle. The longest chord is the diameter.
- Circumference: The distance around the circle is given by \(C = 2\pi r\).
- Area: The area \(A\) of a circle is given by \(A = \pi r^2\).

6. Applications of the Equation of a Circle

The equation of a circle has numerous applications across different fields:

1. Geometry: Used to solve geometric problems involving circles.
2. Physics: In mechanics, the path of an object in circular motion can be described using the circle's equation.
3. Computer Graphics: Circles are fundamental shapes in graphics programming and game design.
4. Engineering: Circles are used in structural design and analysis.
5. Astronomy: The orbits of celestial bodies can be approximated using circular equations.

7. Example Problems and Solutions

To reinforce understanding, let's go through some example problems.

7.1 Example Problem 1

Find the center and radius of the circle given by the equation \((x + 5)^2 + (y - 3)^2 = 25\).

Solution:
- Center: \((-5, 3)\)
- Radius: \(r = \sqrt{25} = 5\)

7.2 Example Problem 2

Convert the equation $x^2 + y^2 - 4x + 8y - 12 = 0$ into standard form.

Solution:
1. Rearranging gives:
$$x^2 - 4x + y^2 + 8y = 12$$
2. Completing the square:
- For x: $x^2 - 4x$ becomes $(x - 2)^2 - 4$
- For y: $y^2 + 8y$ becomes $(y + 4)^2 - 16$

3. Substitute back:
$$(x - 2)^2 - 4 + (y + 4)^2 - 16 = 12$$
$$(x - 2)^2 + (y + 4)^2 = 32$$

The standard form is:
$$(x - 2)^2 + (y + 4)^2 = 32$$

8. Conclusion

The equation of a circle is a fundamental concept that links various areas of mathematics and its applications. Understanding both the standard and general forms allows for the manipulation and analysis of circular shapes in numerous contexts. By mastering the properties, derivation, and applications of the circle's equation, students can enhance their problem-solving skills and apply these concepts effectively in real-world scenarios. The examples provided serve as a guide to help solidify understanding, making it easier to tackle similar problems in the future.

Frequently Asked Questions

What is the standard form of the equation of a circle?

The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

How do you derive the equation of a circle from its center and radius?

To derive the equation, start with the definition of a circle as the set of all points equidistant from a center point. The distance from any point (x, y) to the center (h, k) is r, leading to the equation $(x - h)^2 + (y - k)^2 = r^2$.

What does the variable 'r' represent in the equation of a circle?

'r' represents the radius of the circle, which is the distance from the center (h, k) to any point on the circle.

Can the equation of a circle be expressed in a different form?

Yes, the equation can also be expressed in general form as $x^2 + y^2 + Dx + Ey + F = 0$, where D, E, and F are constants that can be derived from the center and radius.

What is the significance of the center of the circle in its equation?

The center (h, k) determines the location of the circle in the Cartesian plane, and it is the point from which all points on the circle are equidistant.

How can you find the center and radius from the general form of a circle's equation?

To find the center and radius from the general form, $x^2 + y^2 + Dx + Ey + F = 0$, you can complete the square for both x and y to convert it to standard form and identify (h, k) and r.

What are some applications of the equation of a circle in real life?

The equation of a circle is used in various fields such as physics for modeling circular motion, in engineering for designing circular structures, and in computer graphics for rendering circles and arcs.

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