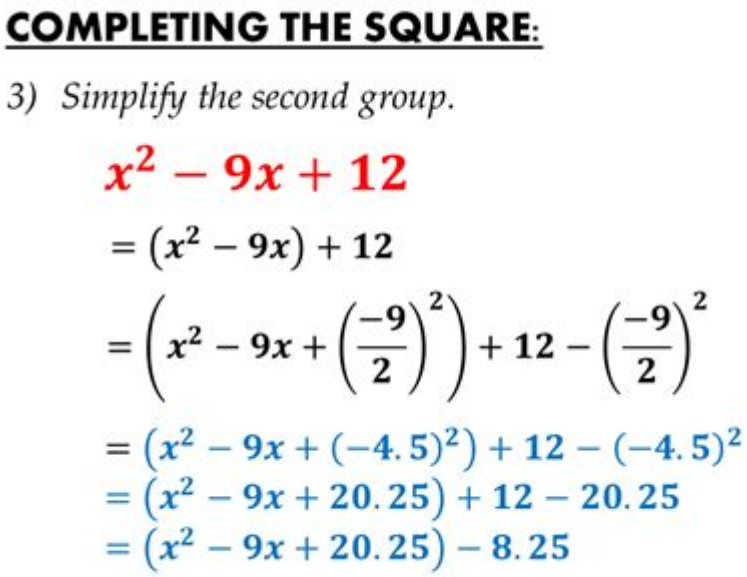
Equations Of Circles Completing The Square Worksheet Answers



Equations of circles completing the square worksheet answers are essential tools in understanding the geometry of circles in a coordinate plane. Completing the square is a method used to rewrite quadratic equations in a form that is more manageable and insightful, particularly when dealing with conic sections like circles. This article will delve into the step-by-step process of completing the square to derive the equation of a circle, the standard form of a circle's equation, and provide example problems along with detailed answers.

Understanding the Circle's Equation

A circle is defined as the set of all points in a plane that are equidistant from a fixed point known as the center. The standard form of the equation of a circle with center \((h, k)\) and radius \(r\) is given by:

\[
(x - h)^2 + (y - k)^2 = r^2
\]

Here, \((h, k)\) represents the coordinates of the center of the circle, and \(r\) represents the radius. To derive this equation from a general quadratic equation, we often need to complete the square.

Completing the Square

Completing the square is a technique used in algebra to convert a quadratic expression into a perfect square trinomial. This is particularly useful for rewriting the equation of a circle from standard quadratic form into standard circle form.

Steps to Complete the Square

1. Start with the general equation of a circle:
$$
Ax^2 + Ay^2 + Bx + Cy + D = 0
$$
Here, A, B, C, and D are constants.

2. Rearrange the equation to group x and y terms:
$$
Ax^2 + Bx + Ay^2 + Cy + D = 0
$$

3. Factor out the coefficient of x^2 and y^2 (if $A \neq 1$):
$$
A(x^2 + \frac{B}{A}x) + A(y^2 + \frac{C}{A}y) + D = 0
$$

4. Complete the square for x:
- Take half of the coefficient of x (which is $\frac{B}{A}$), square it, and add it inside the parenthesis.
- The term added is $\left(\frac{B}{2A}\right)^2$.

5. Complete the square for y:
- Similarly, take half of the coefficient of y (which is $\frac{C}{A}$), square it, and add it inside the parenthesis.
- The term added is $\left(\frac{C}{2A}\right)^2$.

6. Adjust the equation:
- Ensure that the equation remains balanced by subtracting the added squares multiplied by A from D.

7. Write the equation in standard form:
$$
(x - h)^2 + (y - k)^2 = r^2
$$

Example Problems

To illustrate the process of completing the square in the context of circles, let's look at a few

examples.

Example 1

Problem: Convert the equation $x^2 + y^2 - 6x + 4y - 12 = 0$ into standard form.

Solution:

1. Rearrange the equation:
$$
x^2 - 6x + y^2 + 4y = 12
$$

2. Complete the square for x:
- Take half of -6 (which is -3), square it to get 9.
- Add 9 inside the parenthesis, adjusting the equation:
$$
x^2 - 6x + 9 + y^2 + 4y = 12 + 9
$$

3. Complete the square for y:
- Take half of 4 (which is 2), square it to get 4.
- Add 4 inside the parenthesis:
$$
x^2 - 6x + 9 + y^2 + 4y + 4 = 21
$$

4. Rewrite the equation:
$$
(x - 3)^2 + (y + 2)^2 = 21
$$

5. Standard form:
The center is $(3, -2)$ and the radius is $\sqrt{21}$.

Example 2

Problem: Convert the equation $2x^2 + 2y^2 - 8x + 10y - 9 = 0$ into standard form.

Solution:

1. Rearrange the equation and factor out 2:
$$
2(x^2 - 4x + y^2 + 5y) = 9
$$

2. Complete the square for x:

- Half of -4 is -2, and squaring gives 4:
$$
2(x^2 - 4x + 4 + y^2 + 5y) = 9 + 8
$$

3. Complete the square for y:
- Half of 5 is 2.5, squaring gives 6.25:
$$
2(x - 2)^2 + 2(y + 2.5)^2 = 17.25
$$

4. Divide through by 2 to simplify:
$$
(x - 2)^2 + (y + 2.5)^2 = \frac{17.25}{2}
$$

5. Standard form:
The center is $(2, -2.5)$ and the radius is $\sqrt{\frac{17.25}{2}}$.

Conclusion

Understanding how to derive the equations of circles by completing the square is crucial for solving geometry problems in algebra and calculus. The process not only aids in identifying the center and radius of the circle but also reinforces the connection between algebraic and geometric interpretations. By practicing problems and utilizing worksheets focused on completing the square, students can gain proficiency in handling circle equations, ultimately enhancing their overall mathematical skills.

This comprehensive guide provides clear steps, examples, and explanations that can serve as a reference for students and educators alike in mastering the concept of circle equations through the technique of completing the square.

Frequently Asked Questions

What is the standard form of the equation of a circle?

The standard form of the equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and r is the radius.

How do you convert the general form of a circle's equation to standard form?

To convert the general form $Ax^2 + Ay^2 + Bx + Cy + D = 0$ to standard form, complete the square for the x and y terms and rearrange the equation to the form $(x - h)^2 + (y - k)^2 = r^2$.

What is completing the square in the context of circle equations?

Completing the square involves rewriting a quadratic equation in the form of $(x - h)^2$ or $(y - k)^2$. This method is used to find the center and radius of the circle from its equation.

Can you provide an example of a completed square for a circle equation?

Sure! For the equation $x^2 + y^2 - 6x + 4y - 12 = 0$, completing the square gives $(x - 3)^2 + (y + 2)^2 = 25$, indicating a circle with center (3, -2) and radius 5.

Why is it important to find the center and radius of a circle?

Finding the center and radius of a circle is crucial for graphing the circle accurately and for solving geometric problems involving circles, such as determining intersections and distances.

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