

# Differential Equations Final Exam

## Differential Equations 2280

### Sample Final Exam

Thursday, 28 April 2016, 12:45pm-3:15pm, LCB 219

**Instructions:** This in-class exam is 120 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

#### 1. (Quadrature Equation)

Solve for the general solution  $y(x)$  in the equation  $y' = 2 \cot x + \frac{1250x^3}{1+25x^2} + x \ln(1+x^2)$ .

[The required integration talent includes basic formulae, integration by parts, substitution and college algebra.]

Answer:

$$y = 2 \ln(\sin(x)) + \frac{49}{2}x^2 - \ln(1+25x^2) + 1/2(1+x^2)\ln(1+x^2) - 1/2 + C$$

#### 2. (Separable Equation Test)

The problem  $y' = f(x, y)$  is said to be separable provided  $f(x, y) = F(x)G(y)$  for some functions  $F$  and  $G$ .

(a) [75%] Check (X) the problems that can be put into separable form, but don't supply any details.

<input type="checkbox"/> $y' = -y(2xy + 1) + (2x + 3)y^2$	<input type="checkbox"/> $yy' = xy^2 + 5x^2y$
<input type="checkbox"/> $y' = e^{x+y} + e^y$	<input type="checkbox"/> $3y' + 5y = 10y^2$

(b) [25%] State a test which can verify that an equation is not separable. Apply the test to verify that  $y' = x + \sqrt{|xy|}$  is not separable.

Answer:

(a)  $yy' = xy^2 + 5x^2y$  is not separable, but the other three are separable.

(b) Test:  $f_y/f$  not independent of  $x$  implies not separable.

If the equation is separable for all allowed  $x$  and  $y$ , then it is separable on a smaller domain. Let  $f = x + \sqrt{|xy|}$  and assume  $x > 0$  and  $y > 0$ . Then  $f = x + \sqrt{xy} = \sqrt{x}\sqrt{y}$ . We have  $f_y = \frac{1}{2}\sqrt{x}/\sqrt{y}$  and  $f_y/f = \frac{1}{2}\sqrt{x}/(x\sqrt{y} + y\sqrt{x})$ . When  $y = 1$ , then this fraction depends on  $x$ , so the DE is not separable.

Differential equations final exam preparation is a critical step for students pursuing mathematics, engineering, physics, or any field that employs mathematical modeling. Differential equations are equations that relate a function with its derivatives and are essential for modeling dynamic systems in various real-world applications. The final exam in a differential equations course often encompasses a broad range of topics, requiring a solid understanding of both theory and application. This article will provide an overview of the key concepts, types of differential equations, and effective study strategies to help students excel in their final exam.

# Understanding Differential Equations

Differential equations can be classified into several categories, each with distinct characteristics and methods of solution. A basic understanding of these categories is crucial for success in a final exam.

## Types of Differential Equations

### 1. Ordinary Differential Equations (ODEs):

- These involve functions of a single variable and their derivatives.
- Example:  $\left( \frac{dy}{dx} + y = e^x \right)$

### 2. Partial Differential Equations (PDEs):

- These involve functions of multiple variables and their partial derivatives.
- Example:  $\left( \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \right)$

### 3. Linear vs. Nonlinear Differential Equations:

- A linear differential equation can be written in the form  $\left( a_n(x) \frac{d^n y}{dx^n} + a_{n-1}(x) \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_0(x)y = g(x) \right)$ .
- Nonlinear equations cannot be expressed in this form and often require different solution techniques.

### 4. Homogeneous vs. Non-Homogeneous:

- Homogeneous equations have the form  $\left( L(y) = 0 \right)$ , where  $\left( L \right)$  is a linear operator.
- Non-homogeneous equations have the form  $\left( L(y) = g(x) \right)$  with a non-zero function  $\left( g(x) \right)$ .

## Key Concepts and Methods

Understanding the fundamental methods for solving different types of differential equations is essential for the final exam.

- **Separation of Variables:** This technique is often used for first-order ODEs and involves separating the variables into two sides of the equation for integration.

- **Integrating Factor:** For linear first-order ODEs, an integrating factor is used to convert the equation into an exact differential.

- **Characteristic Equation:** For linear second-order homogeneous equations with constant coefficients, the characteristic equation is solved to find the general solution.

- Laplace Transforms: This method is particularly useful for solving linear ODEs and PDEs, especially those with discontinuous or impulsive inputs.
- Numerical Methods: Techniques such as Euler's method or the Runge-Kutta method can be employed to approximate solutions when analytic solutions are complex or infeasible.

## **Exam Preparation Strategies**

Preparing for a differential equations final exam involves a combination of reviewing theoretical concepts, practicing problem-solving skills, and managing time effectively during the exam. Here are some effective strategies:

### **1. Review Course Material**

- Textbooks and Lecture Notes: Go through your textbooks and lecture notes, focusing on key definitions, theorems, and example problems.
- Key Formulas: Create a list of essential formulas and theorems that are often tested, such as the solutions to different types of differential equations.

### **2. Practice Problem Solving**

- Work on Past Exam Papers: Solving previous exams can provide insight into the types of questions that may appear.
- Variety of Problems: Practice a wide range of problems, including both theoretical questions and practical applications.
- Group Study: Collaborate with classmates to discuss solutions and approaches to different problems.

### **3. Utilize Online Resources**

- Video Tutorials: Websites like Khan Academy or YouTube have numerous tutorials on solving differential equations.
- Online Forums: Engaging with online communities such as Stack Exchange can provide additional support and clarification on challenging topics.

### **4. Time Management During the Exam**

- Read Instructions Carefully: Ensure you understand what is being asked

before starting to solve a problem.

- **Allocate Time Wisely:** Divide your available time according to the number of questions and their difficulty. Prioritize problems you feel most confident tackling.
- **Check Your Work:** If time permits, review your answers to catch any mistakes or miscalculations.

## **Common Topics Covered in the Final Exam**

While the specific content of a differential equations final exam may vary, certain topics are frequently emphasized. Understanding these topics will help students focus their studies effectively.

### **1. First-Order Differential Equations**

- Techniques for solving separable equations, linear equations, and exact equations.
- Applications, such as growth and decay models.

### **2. Second-Order Differential Equations**

- Solutions of homogeneous and non-homogeneous equations.
- The method of undetermined coefficients and variation of parameters.

### **3. Systems of Differential Equations**

- Solving systems of linear equations and methods for finding eigenvalues and eigenvectors.
- Applications in population dynamics and coupled oscillators.

### **4. Laplace Transforms**

- Understanding the definition of the Laplace transform and its properties.
- Using Laplace transforms to solve initial value problems.

### **5. Series Solutions and Special Functions**

- Methods for finding power series solutions for differential equations.
- Introduction to special functions such as Bessel and Legendre functions.

## 6. Partial Differential Equations

- Basic techniques for solving first-order PDEs.
- An overview of boundary value problems and Fourier series.

## Conclusion

A differential equations final exam can be a challenging yet rewarding experience for students. By understanding the various types of differential equations, mastering solution techniques, and employing effective study strategies, students can enhance their chances of success. Remember that consistent practice and a solid grasp of the fundamental concepts will prepare you not only for the exam but also for future applications in your academic and professional endeavors. Embrace the challenge, and use the resources available to you to achieve your best performance on the exam day.

## Frequently Asked Questions

### What are differential equations and why are they important in mathematics?

Differential equations are mathematical equations that relate a function to its derivatives. They are important because they model real-world phenomena in fields such as physics, engineering, biology, and economics.

### What topics should I review for my differential equations final exam?

Key topics to review include first-order differential equations, second-order linear differential equations, systems of differential equations, Laplace transforms, and applications of differential equations.

### What is the difference between homogeneous and non-homogeneous differential equations?

A homogeneous differential equation is one where all terms involve the dependent variable or its derivatives, while a non-homogeneous equation includes terms that are independent of the dependent variable.

### How do I solve a first-order linear differential equation?

To solve a first-order linear differential equation, you can use an integrating factor, which is typically  $e^{\int P(x)dx}$ , to make the equation

easier to integrate and find the solution.

## **What is the significance of initial conditions in solving differential equations?**

Initial conditions provide specific values at a given point, which are essential for finding a unique solution to a differential equation among the family of solutions.

## **What role do Laplace transforms play in solving differential equations?**

Laplace transforms are used to convert differential equations into algebraic equations, making them easier to solve, especially for linear ordinary differential equations with constant coefficients.

## **What are some common applications of differential equations in real life?**

Differential equations are commonly used to model population dynamics, heat conduction, fluid flow, electrical circuits, and mechanical systems, among other phenomena.

## **How can I prepare effectively for my differential equations final exam?**

Effective preparation includes reviewing lecture notes, solving practice problems, studying example applications, forming study groups, and utilizing online resources or tutoring for difficult concepts.

## **What are some common mistakes to avoid when solving differential equations?**

Common mistakes include misapplying integration techniques, neglecting initial conditions, forgetting to check the solution against the original equation, and overlooking the significance of discontinuities in piecewise functions.

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