

Calculus 1 Study Guide

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Quick Study ACADEMIC CALCULUS 1

integrals, theory, techniques, sequences

FUNCTIONS, LIMITS AND DERIVATIVES FOR FIRST YEAR CALCULUS STUDENTS

FUNCTIONS

DEFINITIONS

- function.** A correspondence that assigns one value (output) to each member of a given set. The given set of inputs is called the **domain**. The set of outputs is called the **range**. One-variable calculus deals with real-valued functions whose domain is a set of real numbers. If a domain is not specified, it is assumed to include all inputs for which there is a real number output.
- notation.** If a function is named f , then $f(x)$ denotes its value at x , or " f evaluated at x ." If a function gives a quantity y in terms of a variable quantity x , then x is called the **independent variable** and y the **dependent variable**. i ven a function by an equation such as $y = x^2$, one may think of y as shorthand for the function's expression. The notation $x \mapsto x^2$ (" x maps to x^2 ") is another way to refer to the function. The expression $f(x)$ for a function at an arbitrary input x often stands in for the function itself.
- graph.** The graph of a function f is the set of ordered pairs $(x, f(x))$, presented visually with a Cartesian coordinate system. The **vertical line test** states that a curve is the graph of a function if every vertical line meets the curve at most once. An equation $y = f(x)$ often refers to the set of points (x, y) satisfying the equation, in this case the graph of the function f . The **zeros** of a function are the inputs x for which $f(x) = 0$, and they give the x -intercepts of the graph.
- even and odd.** A function f is **even** if $f(-x) = f(x)$, e.g., x^2 ; **odd** if $f(-x) = -f(x)$, e.g., x^3 , or neither.

NUMBERS

- Rational numbers.** A rational number is a ratio p/q of integers p and q , with $q \neq 0$. There are infinite ways to represent a given rational number, but there is a unique "lowest-terms" representative. The set of all rational numbers forms a closed system under the usual arithmetic operations.
- Real numbers.** In this guide, \mathbb{R} denotes the set of real numbers. eal numbers may be thought of as the numbers represented by infinite decimal expansions. ational numbers terminate in all zeros or have a repeating segment of digits. eal numbers that are not rational are called **irrational**. E.g., π , the ratio of circumference to diameter of a circle, is irrational; it may be *approximated* by rational numbers, e.g., $\frac{22}{7}$ and 3.14 .

NEW FROM OLD

- Arithmetic.** The **scalar multiple** of a function f by a constant c is given by $(cf)(x) = c \cdot f(x)$. The sum $f + g$, product fg , and quotient f/g of functions f and g are defined by:
 $(f+g)(x) = f(x) + g(x)$,
 $(fg)(x) = f(x)g(x)$,
 $(f/g)(x) = f(x)/g(x)$.
 In each case, the domain of the new function is the intersection of the domains of f and g , with the zeros of g excluded for the quotient.
- Composition.** If f and g are functions, " f composed with g " is the function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ with domain (strictly speaking) the set of x in the domain of g for which $g(x)$ is in the domain of f . E.g., $\sqrt{1-x^2}$ is the square root composed with $x \mapsto 1-x^2$, with domain $[-1, 1]$.
- Translations.** The graph of $x \mapsto f(x-a)$ is the graph of f translated by a units to the right; e.g., $(a, f(b))$ would be on the graph. The graph of $x \mapsto f(x)+b$ is the graph of f translated b units upward.



- Inverses.** An **inverse** of a function f is a function g such that $g(f(x)) = x$ for all x in the domain of f . A function f has an inverse if and only if it is

ELEMENTARY ALGEBRAIC FUNCTIONS

- Constant and Identity.** A constant function has only one output: $f(x) = c$. The identity function is: $x \mapsto x$, or $f(x) = x$.
- Absolute value.** $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$. The above is an example of a **piecewise definition**. For any x , $\sqrt{x^2} = |x|$.
- Linear functions.** For a linear function, the difference of two outputs is proportional to the difference of inputs. The proportionality constant, i.e., the ratio of output difference to input difference

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
 is called the **slope**. The slope is also the change in the function per unit increase in the independent variable. The linear function

$$f(x) = mx + b$$
 has slope m and y -intercept $f(0) = b$, the graph is a straight line. The linear function with value y_0 at x_0 and slope m is

$$f(x) = y_0 + m(x - x_0)$$
.
- Quadratics.** These have the form

$$f(x) = ax^2 + bx + c \text{ where } (a \neq 0).$$
 The **normal form** is $f(x) = a(x-h)^2 + k$. One has $h = -b/(2a)$ and $k = f(h)$. The graph is a parabola with vertex (h, k) , opening up or down accordingly as $a > 0$ or $a < 0$, and symmetric about the vertical line through vertex. A quadratic has two, one, or no zeros accordingly.

functions continued next page

Calculus 1 Study Guide

Calculus 1, often referred to as differential calculus, is a foundational course in mathematics that deals with the study of rates of change and the slopes of curves. This study guide is designed to help students navigate the essential concepts, techniques, and applications of calculus. The material covered in this guide will provide a comprehensive overview and serve as a valuable resource for understanding the core principles that will be encountered in a typical Calculus 1 course.

Understanding Limits

One of the cornerstones of calculus is the concept of limits. Limits allow us to understand the behavior of functions as they approach specific points or infinity.

Definition of Limits

- Limit of a function: The limit of a function $f(x)$ as x approaches a value a is the value that $f(x)$ approaches as x gets arbitrarily close to a . It is denoted as:

$$\lim_{x \rightarrow a} f(x) = L$$

- One-sided limits: These consider the behavior of the function as it approaches from one side:
- Left-hand limit: $\lim_{x \rightarrow a^-} f(x)$
- Right-hand limit: $\lim_{x \rightarrow a^+} f(x)$

Techniques for Finding Limits

1. Direct Substitution: Substitute the value of a directly into $f(x)$.
2. Factoring: Factor the expression and simplify before substituting.
3. Rationalization: Multiply by a conjugate to eliminate radicals.
4. L'Hôpital's Rule: If you encounter an indeterminate form like $0/0$, take the derivative of the numerator and the denominator.

Limit Properties

- Sum Rule: $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- Product Rule: $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- Quotient Rule: $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ if $\lim_{x \rightarrow a} g(x) \neq 0$

Understanding Derivatives

The derivative measures how a function changes as its input changes. It represents the slope of the tangent line to the curve at a given point.

Definition of the Derivative

The derivative of a function $f(x)$ at a point a is defined as:

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

This formula captures the notion of instantaneous rate of change.

Derivative Rules

1. Power Rule: If $f(x) = x^n$, then $f'(x) = nx^{n-1}$.
2. Constant Rule: If $f(x) = c$ (where c is a constant), then $f'(x) = 0$.
3. Sum Rule: $(f + g)' = f' + g'$
4. Difference Rule: $(f - g)' = f' - g'$
5. Product Rule: $(fg)' = f'g + fg'$
6. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Common Derivatives

Here are some common derivatives you should memorize:

- $f(x) = \sin(x) \rightarrow f'(x) = \cos(x)$
- $f(x) = \cos(x) \rightarrow f'(x) = -\sin(x)$
- $f(x) = e^x \rightarrow f'(x) = e^x$
- $f(x) = \ln(x) \rightarrow f'(x) = \frac{1}{x}$

Applications of Derivatives

Derivatives have various applications, including:

1. Finding Tangent Lines: The slope of the tangent line to the curve at a point is given by the derivative at that point.
2. Optimization Problems: Derivatives are used to find local maxima and minima of functions by setting $f'(x) = 0$.
3. Motion Analysis: In physics, the derivative of the position function gives the velocity function.

Understanding Integrals

Calculus is not only about derivatives; it also revolves around integrals, which can be understood as the accumulation of quantities.

Definition of the Integral

The definite integral of a function $f(x)$ from a to b is defined as:

$$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$

where $\Delta x = \frac{b-a}{n}$ and x_i^* is a sample point in each subinterval.

Fundamental Theorem of Calculus

This theorem connects differentiation and integration:

1. First Part: If F is an antiderivative of f on $[a, b]$, then:

$$\int_a^b f(x) \, dx = F(b) - F(a)$$

2. Second Part: If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t) \, dt$ is continuous on $[a, b]$ and differentiable on (a, b) , with $F'(x) = f(x)$.

Techniques of Integration

1. Substitution: Useful for integrating composite functions.
2. Integration by Parts: Based on the product rule for differentiation.
3. Partial Fractions: Useful for integrating rational functions.

Common Integrals

Here are some integrals that are commonly encountered:

- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
- $\int e^x \, dx = e^x + C$
- $\int \sin(x) \, dx = -\cos(x) + C$
- $\int \cos(x) \, dx = \sin(x) + C$

Conclusion

This Calculus 1 study guide provides a comprehensive overview of essential topics including limits, derivatives, and integrals. Mastering these concepts requires both understanding the theoretical aspects and applying them through practice problems. It's crucial to work through exercises, review derivative and integral rules, and apply these techniques to various problems to solidify your understanding. With diligent study and practice, you'll be well-prepared to tackle the challenges of calculus and appreciate its applications in mathematics, science, and engineering.

Frequently Asked Questions

What are the main topics covered in a Calculus 1 study guide?

A Calculus 1 study guide typically covers limits, derivatives, continuity, the definition of a derivative, differentiation rules, applications of derivatives, and introductory topics on integrals.

How can I effectively use a Calculus 1 study guide to prepare for exams?

To effectively use a study guide, start by reviewing each topic systematically, practice problems related to each concept, utilize visual aids like graphs, and take practice exams to assess your understanding.

What are some common pitfalls students encounter when studying Calculus 1?

Common pitfalls include misunderstanding the concept of limits, struggling with the application of derivative rules, neglecting to practice enough problems, and not fully grasping the geometric interpretations of calculus concepts.

Are there any recommended resources to supplement a Calculus 1 study guide?

Yes, recommended resources include online platforms like Khan Academy, MIT OpenCourseWare, and Coursera, as well as textbooks such as 'Calculus' by James Stewart and 'Calculus: Early Transcendentals' by Howard Anton.

What is the best way to practice derivatives for Calculus 1?

The best way to practice derivatives is to work through a variety of problems, including finding derivatives of polynomial, trigonometric, exponential, and logarithmic functions, and applying the product, quotient, and chain rules in different scenarios.

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